Iranian Journal of Management Studies (IJMS)
Vol.6, No.1, January 2013
pp: 7-25

Application of Monte Carlo Simulation in the Assessment of European Call Options

Hamid Shahbandarzadeh[1*], Khodakaram Salimifard[2], Reza Moghdani[3]

1, 2. Assistant Professor of Management Science, Department of Industrial Management, Persian Gulf University, Bushehr, Iran
3. Master of Science in Operations Research, Department of Industrial Management, Persian Gulf University, Bushehr, Iran

(Received: 10 December 2011; Revised: 17 November 2012; Accepted: 29 December 2012)

Abstract

In this paper, the pricing of a European call option on the underlying asset is performed by using a Monte Carlo method, one of the powerful simulation methods, where the price development of the asset is simulated and value of the claim is computed in terms of an expected value. The proposed approach, applied in Monte Carlo simulation, is based on the Black-Scholes equation which generally defined the pricing of European call options in a dynamic environment. Therefore, the main goal of this study is how can Monte Carlo be applied to finance? Although it is stated that because of being based on randomness, the Monte Carlo method has its obvious disadvantages and does not yield solutions for all possible stock prices, by applying Black-Scholes formula, it is efficient to use this method for calculating payoff. Hence, in the matter of this paper, we introduce the Black-Scholes model and Monte Carlo simulations as main tools to determine.

Keywords:

European call options, Monte Carlo simulation, Black-Scholes formula.

---

* Corresponding Author, Tel: +98-9123396721 Email: shahbandarzadeh@pgu.ac.ir

Introduction

Perceived value of products and services is the most significant factor in price setting. Hence, companies design marketing campaign to influence customer’s perception of the product value. When an investor sells or buys stocks on the Internet, sometimes a difference exists between the actual price and the theoretical value (Yoshida, 2003). To manage these differences, managers use different strategy for pricing, In addition to setting the price level, managers have the opportunity to design innovative pricing models that make a balance between customers’ perception and the company requirements.

Option price theory has many applications in finance (Zmeskal, 2001) and gives the buyer the opportunity to purchase a quantity of assets at a certain price within a certain time. According to Linde & Aberg (2004), a European call option is a contract between buyer and seller with the following three conditions:

- The holder of the option has at a prescribed time in future, the expiry date (exercise date), and the right to buy a prescribed asset. The asset is usually a stock which is bought for a prescribed amount, the exercise price (strike price).
- The holder of the option is in no way obliged to buy the underlying asset.
- The exercise price and the expiry date are determined at the time when the option is written.

Although European call option is widely used in many contexts, according to Maidanov (2010), there are some limitations involved in the Black-Scholes model as following assumptions:

- The cost is not constant, therefore, the liquidation of market is assumed so that there are no transaction costs.
- The market price is just changed continuously according to one model, specifically the geometric Brownian motion process.
- Underlying security is perfectly divisible, and short selling with full use of proceeds is possible.
- The trading process is assumed as a continuous process.
- Constant risk-free rates are assumed.

- The principle of no arbitrage is assumed to be satisfied.

Another type of options, which plays an important role in financial market, is American options. For an American call option, sometimes termed Bermudan derivatives (Laprise, Fu, Marcus, Lim, & Zhang, 2005), the right to buy the underlying asset can be exercised at any time before the expiry date, not only at the expiration date which is the case for European options. Also, the pricing of European options is a comparatively less difficult task than American call options (Laprise et al., 2005). Many of the researchers, such as Carr, Jarrow, and Myeni (1992), Kim (1990), Broadie and Detemple (1996), Amin (1993), Zhang, Das (1997), Huang, Subrahmanyam and Yu (1996), focus on American call options and present the most popular pricing methods. The European/American classification has nothing to do with the continent (mainland Europe) of origin; it refers specially to the technicality of time the option can be exercised. A put option gives the holder the right to sell a share of the underlying asset, in the same way as a call option gives the holder the right to buy the underlying asset. In an analogous manner, the owner of a put option wants the asset price to fall as low as possible.

One of the most challenging derivatives in finance is the pricing of American call options, sometimes termed Bermudan derivatives, which is defined as to be derivative contracts with early exercise opportunities at a finite number of exercise dates prior to expiration. The major difficulty in pricing such derivatives with “early exercise” features exists in the early-exercise policies.

In this case, main discussion will be in the context of European call options. This paper is organized as follows. Literature review is described in Section 2. The models and Black-Scholes equation are depicted in Sections 3 and 4. The Monte Carlo simulations, using random numbers, are presented in Section 5, and finally, the paper finishes with a discussion in Section 6.

Literature Review

While the solution of the European call option was extracted by

Black and Scholes (1973), researchers present many methodologies for the option pricing based on modification of this formula. So, we begin by reviewing some of the non-simulation-based literature on pricing European options in order to put this paper in the context. Inspired by Rompolis (2010), various methods were recommended in the literature to retrieve the correct risk from option prices and, hence, explain the empirical failures of the Black-Scholes model. Levy (1992) developed a simple methodology that yields closed-form analytical approximations for valuing European option claims involving the arithmetic average of future foreign exchange rates. Wu (2004) proposed the application of fuzzy sets theory to the Black-Scholes formula.

There is a problem to give a meticulous estimate of this volatility. Muzzioli and Toricelli (2001) handle this problem by using possibility distributions. Reynaerts and Vanmele (2003) present an alternative solution to the problem by performing a sensitivity analysis for the pricing of the option. Andersen and Damgaard (1999) suggest an approach to compute the reservation price of an option in an economy with more than one risky security and where trade involves transaction costs.

Cox et al. (1979) presented a discrete time version of the Black-Scholes model, where the option price converges to the Black-Scholes price if the number of trading periods goes to infinity. Rompolis (2010) recommended a new method of implementing the principle of maximum entropy to retrieve the risk neutral density of future stock, or any other asset, returns from European call and put prices.

Reynaerts et al. (2006) focus on Chalasani model, and notice that the intervals of this model do not always lie within the Black-Scholes intervals. They have proved that the bounds they have defined converge to the corresponding bounds in the Black-Scholes model. Their numerical illustrations also show that the hedging error is small if the Asian option is in the money.

It is known that the option pricing theory has many applications in finance. Many researchers, therefore, have applied a great deal of modifications to the Black-Scholes model with different process. Also, the term “real option” has been used in many European call option contexts. It is believed (Zmeskal, 2001) that the real option concept could help understanding the European call option. There are many researchers who used different models of European call option in their works. The main studies of this issue are listed in following table.

Table 1: The main studies of European call option

	Authors	Reference	Description
1	Chalasani et al.	(Chalasani, Jha, & Varikooty, 1998)	It determines accurate lower and upper bounds for the price of a European-style Asian option.
2	Rogers and Shi	(Rogers & Shi, 1995)	It determines lower and upper bounds for the price of a European-style Asian option.
3	Kaas et al.	(Kaas, Dhaene, & Goovaerts, 2000)	It uses random variables to determine bounds of call options.
4	Black and Scholes	(Black & Scholes, 1973)	It presents basic principle of pricing European options.
5	Rompolis	(Rompolis, 2010)	It designs new tools to estimate the risk neutral density (RND) of asset returns.
6	Zmeskal	(Zmeskal, 2001)	It applies a new fuzzy stochastic model to valuing European call option.
7	Andersen and Damgaard	(Andersen & Damgaard, 1999)	It suggests an approach to compute the reservation price of an option in an economy with multiple risks.
8	Wang	(Wang, 2011)	It models discrete time option pricing using the fractional Black-Scholes model with transaction costs.
9	Malz	(Malz, 1996)	It uses jump-diffusion model for estimating the realignment probabilities of option pricing.
10	Xu et al.	(Xu, Li, & Zhang, 2010)	I proposes a fuzzy model based on Greek letters.
11	Garman-Kohlhagen model	(Garman & Kohlhagen, 1983)	Proposing the closed-form solution of a European currency options pricing model.
12	Yoshida	(Yoshida, 2003)	It evaluates European options in uncertain environment based on randomness and fuzziness.
13	Andreou et al.	(Andreou, Charalambous, & Martzoukos, 2008)	It combines artificial neural networks and parametric models in order to pricing European options.
14	Cassidy et al.	(Cassidy, Hamp, & Ouyed, 5736–5748)	It presents a Gosset formula for pricing European options with a log Student's t-distribution.
15	Levy	(Levy, 1992)	It analyses the sensitivity of pricing European options.

Models

Black-Scholes partial differential equation (PDE), which can be used to determine the price of an option, was presented in 1973 by Fischer Black and Myron Scholes (2004). In their well-known work, Black and Scholes converted the option pricing problem into the task of solving a (parabolic) partial differential equation (PDE) with a final condition. Although in the last three decades, the fact that Black-Scholes and its variants are regarded as the most eminent achievements in financial theory, some empirical research has disclosed that the formula suffers from systematic biases (Andreou, Charalambous, & Martzoukos, 2008; Bates, 2003; Bakshi, Cao, & Chen, 1997). The bias stems from the fact that the model has been developed under a set of simplified assumptions such as geometric Brownian motion of stock price movements, constant variance of the underlying returns, continuous trading on the underlying asset, constant interest rates, and so on.

There are different types of modes which have proved the Black-Scholes equation. To introduce the effective way, we use these modes simultaneously. To derive the Black-Scholes equation, we apply the Itô's lemma (Levy, 1992), which is the basic assumption of a geometric Brownian motion for the asset price. Actually the most important application of the Itô's calculus, derived from the Itô's lemma, in financial mathematics is the pricing of options. Assuming that the asset price is S, the geometric Brownian motion is as follows (Wang, 2011):

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{1}$$

Where S is the spot price of the underlying asset and μ is the drift rate of S, which annualized and ,σ is the standard deviation of continuous returns and W is Brownian motion (originally is a physic concept). In this formula, μ and σ are constant. Let V=V(S, t) denote the value of an option as function of S and t. Also, Itô's lemma respected dV as follows:

$$dV = \left(\mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \frac{\partial V}{\partial S} d \tag{2}$$

Now considering a certain portfolio, sometimes called the delta-hedge portfolio, consisting of being short one option and long shares at time *t*. The value of these holdings is as follows:

$$\Pi = -V + \Delta S \tag{3}$$

Where ∏ is the value of a portfolio and value *V* and Δ as units of the underlying asset with the price. Using its formula, we obtain the change in the value of portfolio as:

$$d\Pi = -dv + \Delta ds = \left(\mu S \left[\Delta - \frac{\partial V}{\partial S} \right] + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \left(\frac{\partial V}{\partial S} \right. \quad 4)$$

Hence, by following assumption:

$$\Delta = \frac{\partial V}{\partial S} \tag{5}$$

The infinitesimal change $d\prod$ of the portfolio within the time interval *dt* is

$$d\Pi = -\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt \tag{6}$$

And, stated that it is completely deterministic, so, the effect of drift rate μ has been eliminated. This represents the gain when, $\prod_0$ the initial wealth, is invested in the risky, but frictionless market (which means all parties have immediate access to any information, and all securities and credits are available at any time and in any size) which consists of the option with value *V* and the underlying asset with *S*. Under the assumption of a frictionless market without arbitrage and a constant risk-free interest rate *r*, hence, the change in infinitesimal time as follows:

$$\Delta\Pi = r\Pi dt \tag{7}$$

According to Formula (6) and (7), consequently, relation calculated as follows:

$$\left(-\frac{\partial V}{\partial t}-\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta t = r\left(-V+\frac{\partial V}{\partial S}S\right)\Delta \tag{8}$$

Hence, for European options the partial differential according to Relation (8), Black-Scholes equation can be presented as following function:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}-rV+rS\frac{\partial V}{\partial S}=0 \tag{9}$$

Black-Scholes Formula

As stated before, to calculate the price of European put and call options, we apply the Black-Scholes formula. It can be gained by solving the Black–Scholes stochastic differential equation for the corresponding expressed in the previous section, so the well-known Black–Scholes formula for European call option written on a stock *S* with expiry date *T* and strike price *K* is described as follows. Let the function *C* be given by the formula (Panayiotis, Andreou, Charalambous, & Martzoukos, 2008)

$$C(S,t)=N(d_1)S-N(d_2)Ke^{-r(T-t)} \tag{10}$$

Where

$$d_1=\frac{\ln(\frac{S}{K})+(r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{11}$$

Where σ implies volatilities over time. In fact, it is one of the difficulties in computing call option payoff. That is why it has been simply ignored in many articles.

And, d_2 be given as follows:

$$d_2=\frac{\ln(\frac{S}{K})+(r-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{12}$$

Also,

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{13}$$

Finally, the price of a European call option (payoff) is given (Adams & Wyatt, 1987; Rompolis, 2010):

$$EOC = Max(C-K,0)$$

For put options a similar process is used with expected payoffs. Using its formula, the price of a corresponding put option based on put-call parity is calculated using Equation 14.

$$p(S,t) = Ke^{-r(T-t)} - S + C(S,t) \tag{14}$$

Also,

$$dS(t) = (\mu - q)S(t)dt + \sigma S(t)dW(t) \tag{15}$$

We can use Equation 15 to derive the price of the European call option with referring these studies (Zmeskal, 2001).

Monte Carlo Simulation

In this paper, the pricing of a European call option is carried out using a Monte Carlo method, where the price development of the asset is simulated based on the generation of random numbers. A call option, a widely employed contract between a buyer and a seller, is financial asset that can be similar to a lottery coupon. Monte Carlo analysis is a statistical technique that could become progressively prominent as a tool for risk estimator to evaluate the uncertainty (Rezaie, Amalnik, Gereie, & Shakhseniaee, 2007). In spite of the fact that Monte Carlo simulation has been utilized since the 1940s, more effective desktop computers have accomplished its accessible and attractive simulation for many new applications (Jian-wen & Xing-xia, 2009).

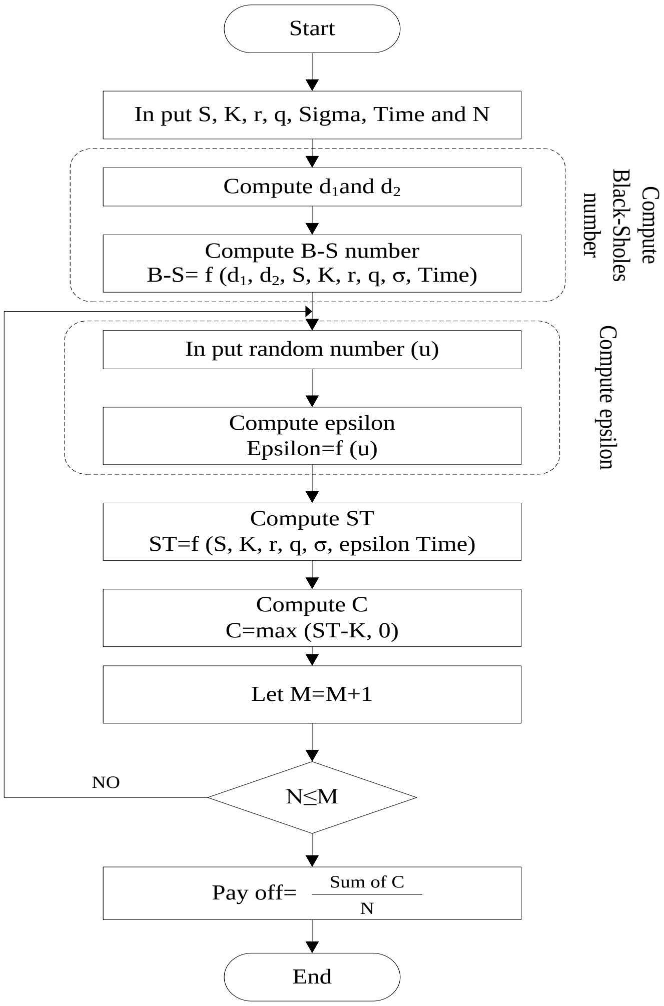

Figure 1: The flowchart of proposed Monte Carlo Simulation

In a Monte Carlo test, a large number of simulations are carried out simultaneously. Starting from today's asset value, random changes are added, one for each step in the time. When having reached the exercise date, possible future values of the asset are obtained, and from that the corresponding option prices are computed according to the termination condition (in this case *max*(*ST*-*K*, 0)). Finally, a single mean value of the results is computed and discounted to today's date. The flowchart of the proposed Monte Carlo simulation is depicted in Figure 1.

This model is coded in Excel VBA which is a programming language and it has been used widely by programmers recently. The model, therefore, takes in six parameters, *S* is the price of the share at the initial time, *K* is strike price, *r* is the domestic risk-free interest rate, *q* is the risk-free interest rate of the foreign security, *Time* is expiration date, σ is the volatility of the rate, *N* is the number of simulation runs.

Results

According to Klar and Jacobson (2002), in order to prove the merit of Monte Carlo method, the standard error must be considered as the main tool to determine the validity of the model, where the model is based on random numbers. The statistical error for a general Monte Carlo method is $\frac{1}{\sqrt{N}}$, where *N* is the number of simulations. To calculate the statistical error, we used an implementation of the following formula (16). In this formula, only one time step is needed, so the time discretization of the error is totally eliminated.

$$S_t = S_{t-1} e^{((r-\frac{1}{2}\sigma^2)\Delta t+\sigma W)} \tag{16}$$

The two following graphs illustrate the convergence of the exponential Monte Carlo method and the statistical error in two cases with different expiration dates. As it can be seen in Figure 1, it is hard to discover a pattern in the successive errors. In fact, it is obvious that the Monte Carlo errors converge toward the reference errors. This behavior is aroused from a randomness characteristic of Monte Carlo method.

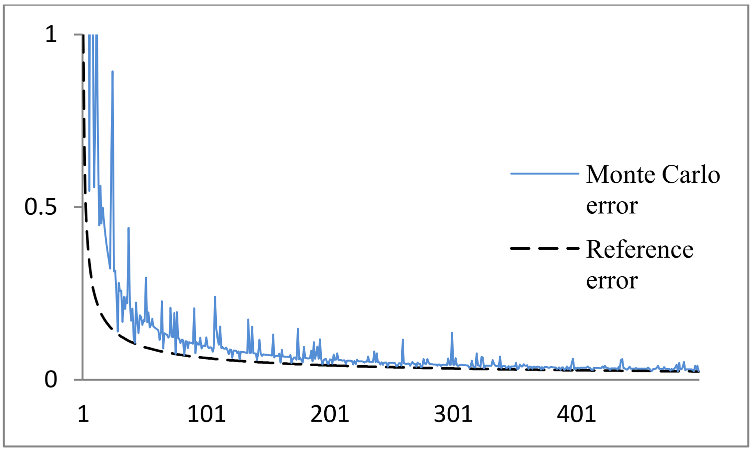

Figure 2: Convergence of standard error in T=0.25

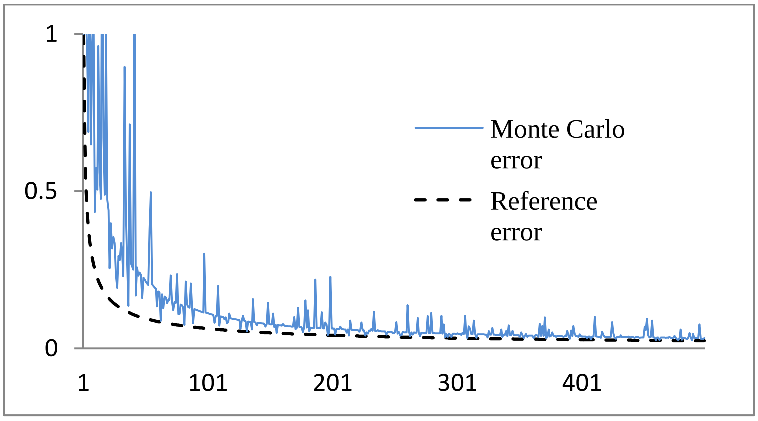

Figure 3: Convergence of standard error in T=0. 5

To compute the expected payoff, the interest rate r is set to (5%) (r=.05), the strike price is K = 56, and the exercise date T is set to one year. As mentioned before, it would be erroneous if one supposed volatilities to be constant over time. Hence, different values for volatilities (σ) are used during simulation:

$$\sigma = \begin{cases} 0.1 & t \in (today, \frac{T}{4}] \\ 0.2 & t \in (\frac{T}{4}, \frac{T}{2}] \\ 0.4 & t \in (\frac{T}{2}, \frac{3T}{4}] \\ 0.8 & t \in [\frac{3T}{4}, T] \end{cases}$$

Therefore, with regard to different volatilities, which have been aroused in different times, the proposed model is used to compute upper and lower bounds of expected payoff. The results are presented in Table 1.

Table 1. Results of the model

	Down bounds of EP	Upper bounds of EP	Standard error	Average ST
σ=0.1	0	2.768	0.254	54.821
σ=0.2	2.768	4.255	0.743	56.223
σ=0.4	4.255	5.227	0.954	55.711
σ=0.8	5.227	7.448	1.918	57.021

The resulting convergence of expected payoff is depicted in Figure 4. With different volatilities, we included expected payoff converged at the best solution. As it is shown, in iteration 500, value of payoffs converges with different volatilities at that point. That is, this graph indicates that Monte Carlo simulation is generally performing efficiently for European call options.

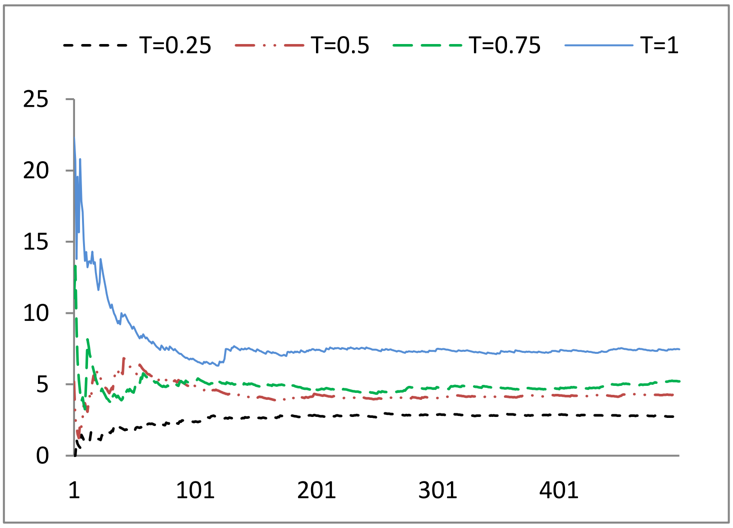

Figure 4: Expected payoff diagram obtained from Monte Carlo simulation

Although Monte Carlo method does not provide solutions for all stock prices (Klar & Jacobson, 2002), but using this method, the great fluctuation that may occur in stock prices can be scientifically discovered.

Conclusion

Many studies suggest that the Monte Carlo method can be used to obtain numerical estimates of European call option on stock which pays discrete dividends. This method has some strengths and limitations where its usage would be advantageous. With regard to the distribution used to generate the returns on the underlying stock, flexibility is one of the main advantages of the Monte Carlo method. Changing the underlying distribution merely involves using a different process for generating random varieties employed in the method. Furthermore the Monte Carlo method is perhaps unique in the sense that the distribution used to generate returns on the underlying stock need not have a closed-form analytic expression.

This study wants to show how the pricing of a European call option is computed by applying Monte Carlo Simulation. We presented a new approach to pricing European-style derivatives through approximating the value function with nonlinear functions. To illustrate the approach, we applied our algorithms to European call and put options written on underlying assets following geometric Brownian motion and jump-diffusion processes. The problem stated in computing option market prices is solved by applying Black-Scholes formula. Because of being based on randomness, the Monte Carlo method has its obvious disadvantages method and does not yield solutions for all possible stock prices, but by applying Black-Scholes formula, it is efficient to use this method for calculating payoff. In a Monte Carlo test, a large number of simulations are carried out simultaneously; starting from today's asset value, random changes are added, one for each time step in the time. When having reached the exercise date, possible future values of the asset are obtained, and from that the corresponding option prices are computed according to the terminal condition.

There are many other researches (Carr et al. 1992; Maidanov, 2010; Rompolis, 2010) which conceivably indicate that using the Monte Carlo method, valid results are expected in pricing simulation. According to Klar and Jacobson (2002), computed standard error of pricing European call options can be used to prove the merit of Monte Carlo method. They used a graph to illustrate the convergence between exponential Monte Carlo method and standard error of pricing European call options. Our findings are highly consistent with their results. This ensures us that the results are reliable and can be used in making right decision regarding to European call options.

Acknowledgements

The authors would like to sincerely thank the anonymous referees for their constructive criticism, valuable advice, and many helpful comments. Their suggestions led to substantial improvements of the paper.

References

Adams, P., & Wyatt, S. (1987). On the pricing of European and American foreign currency call options. *Journal of Internatinonal Money and Finance, 6*, 315-328.

Amin, K. (1993). Jump dffiusion option valuation in discrete time. *Journal of Finanace, 48*(5), 1833-1863.

Andersen, E., & Damgaard, A. (1999). Utility based option pricing with proportional transaction costs and diversification problems:An interior-point optimization approach. *Applied Numerical Mathematics, 29*(3), 395-422 .

Andreou, P.C., Charalambous, C., Martzoukos, S. H. (2008). Pricing and trading European options by combining artificial neural networks and parametric models with implied parameters. *European Journal of Operational Research, 185*, 1415-1433.

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical performance of alternative options pricing models. *Journal of Finance,* 2003-2049.

Bates, D. (2003). Empirical option pricing: A retrospection. *Journal of Econometrics*, 387-404.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*, 637-659.

Broadie, M., & Detemple, J. (1996). American option valuation: New bounds, approximations, and a comparison of existing methods. *Review of Finance, 9*(4), 1211-1250.

Carr, P., Jarrow, R., & Mynemi, R. (1992). Alternative characterizations of American put options. *Mathematical Finance*, 87-106.

Cassidy, D., Hamp, M., & Ouyed, R. (5736–5748). Pricing European options with a log Student's t-distribution: A Gosset formula. *Physica A*, 2010.

Chalasani, P., Jha, S., & Varikooty, A. (1998). Accurate approximations for European-style Asian options. *Journal of Computational finance*, 11-30.

Cox, J., Ross, S., & Rubinstein, M. (1979). Options pricing: A simplified approach. *Journal of Financial Economics*, 229-263.

Das, S. (1997). Discrete-time bond and option pricing for jump- Diffusion processes. *Review of Derivatives Research*, (3), 211-244.

Garman, M., & Kohlhagen, S. (1983). Foreign currency option values. *Journal of International Money and Finance*, 231-237.

Huang, J., Subrahmanyam, M., & Yu, G. (1996). Pricing and hedging American options: a recursive integration method. *Review of Finance, 9*(1), 277-300.

Jian-wen, H., & Xing-xia, W. (2009). Risk analysis of construction schedule based on Monte Carlo simulation . *IEEE*, 56-60.

Kaas, R., Dhaene, J., & Goovaerts, M. (2000). Upper and lower bounds for sums of random variables. *Insurance: Mathematics & Economics, 27,* 151-18.

Kim, I. (1990). The analytical valuation of American options. *Review of Finance, 3*(4), 547-572.

Klar, L., & Jacobson, J. (2002). *Pricing of European Call Options.* Uppsala: Uppsala University.

Laprise, S., Fu, M., Marcus, S., Lim, A., & Zhang, A. (2005). *Pricing American-Style derivatives with European call options.* National Science Foundation.

Levy, E. (1992). Pricing European average rate currency options. *Journal of International Money and Finance, 11*, 474-491.

Linde, G., & Aberg, M. (2004). *Pricing European call options using a pseudospectral method.* Uppsala: Report in Scientific computing advanced course .

Linde, G., & Aberg, M. (2004). *Pricing European call options using a pseudospectral method.* Uppsala: Uppsala University.

Maidanov, S. A. (2010, Nov 01). *Monte Carlo European options pricing implementation using various industry library solutions*. Newyork City, USA: *INTEL*.

Malz, A. (1996). Using option prices to estimate realignment probabilities in the European Monetary System: The case of sterling-mark. *Journal of International Money and Finance, 15*(5), 717-748.

Muzzioli, C., & Torricelli, C. (2001). A multiperiod binomial model for pricing options in an uncertain world. *Proceedings of the Second International Symposium on Imprecise Probabilities and Their Applications*, 255-264.

Panayiotis, D., Andreou, C., Charalambous, C., & Martzoukos, S. (2008). Pricing and trading European options by combining artificial neural networks and parametric models with implied parameters. *European Journal of Operational Research*, *185*, 1415-1433.

Reynaerts, H., & Vanmaele, M. (2003). A sensitivity analysis for the pricing of European call options in a binary tree model. *Proceedings of ISIPTA*, 1-15.

Reynaerts, H., Vanmaele, M., Dhaene, J., & Deelstra, G. (2006). Bounds for the price of a European-style Asian option in a binary tree model. *European Journal of Operational Research*, 322-332.

Rezaie, k., Amalnik, M., Gereie, A., B, O., & Shakhseniaee, M. (2007). Using extended Monte Carlo simulation method for the improvement of risk management: Consideration of relationships between uncertainties. *Applied Mathematics and Computation*, *190*, 1492-1501.

Rogers, L., & Shi, Z. (1995). The value of an Asian option. *Journal of Applied Probability, 32*, 1077-1088.

Rompolis, L. (2010). Retrieving risk neutral densities from European option prices based on the principle of maximum entropy. *Journal of Empirical Finance, 17*, 918-937.

Wang, X. (2011). Scaling and long-range dependence in option pricing V: Multiscaling hedging and implied volatility smiles under the fractional Black-Scholes model with transaction costs. *Physica A, 390*, 1623-1634.

Wu, H. -C. (2004). Pricing European options based on the fuzzy pattern of Black-Scholes formula. *Computers & Operations Research, 31*, 1069-1081.

Xu, W., Li, H., & Zhang, W. (2010). A study of Greek letters of currency option under uncertainty environments. *Mathematica land Compute rModelling*.

Yoshida, Y. (2003). The valuation of European options in uncertain environment. *European Journal of Operational Research*, *145*, 221-229.

Zmeskal, Z. (2001). Apllication of the fuzzy-stochastic methology to appraising the firm value as a Eroupian call option. *European Journal of Operational Reaserch*, *135*(2), 303-310.