

Optimization of the Inflationary Inventory Control Model under Stochastic Conditions with Simpson Approximation: Particle Swarm Optimization Approach

Seyed Mostafa Orand¹, Abolfazl Mirzazadeh^{2*}, Farzaneh Ahmadzadeh³, Farid Talebloo⁴

1. Faculty of Industrial Engineering, Islamic Azad University, Science and Research Branch, Saveh, Iran

2. Faculty of Industrial Engineering, Kharazmi University, Tehran, Iran

3. Faculty of Industrial Engineering, Islamic Azad University, Karaj Branch, Iran

4. Department of Information Technology of Sufi Razi, Zanjan, Iran

(Received: 17 September, 2014; Revised: 1 January, 2015; Accepted: 5 January, 2015)

Abstract

In this study, we considered an inflationary inventory control model under non-deterministic conditions. We assumed the inflation rate as a normal distribution, with any arbitrary probability density function (pdf). The objective function was to minimize the total discount cost of the inventory system. We used two methods to solve this problem. One was the classic numerical approach which turned out to be prohibitively difficult. The other was a proposed combination method which used Simpson approximation and particle swarm optimization (PSO). To illustrate the theoretical results, we have provided numerical examples.

Keywords

Inflation, Inventory systems, Particle swarm optimization, Simpson approximation, Stochastic.

* Corresponding Author, Email: a.mirzazadeh@aut.ac.ir

Introduction

In recent years, a large number of studies have focused on inventory management systems. For the majority of these studies, one of two procedures have been used. The first procedure determines the optimal values of the decision system variables by minimizing the average annual cost. The second (and, in theory, more correct) procedure determines the optimal ordering policy by minimizing the discounted value of all future costs (Mirzazadeh, 2007, pp. 658-666). Thus, each model has a different scope, parameter and method for solving. The problem is finding the time performance of the proposed method for solving the complex inventory models in order to determine the optimal decision parameter. In this study, we used an inflationary inventory model under stochastic condition with two unknown parameters. These parameters were (k) proportion of time in any given inventory cycle and (T) replenishment time inventory.

Figure 1 shows the scope and parameters of the inventory models with the most deteriorating items (Li & Mawhinney, 2010). In this study, we considered these models with the inflation rate. It is important to note that the rate can be deterministic or stochastic. After describing the resulting models, we considered the methods to solve them. We proposed a combination method using Simpson's rules-particle swarm optimization.

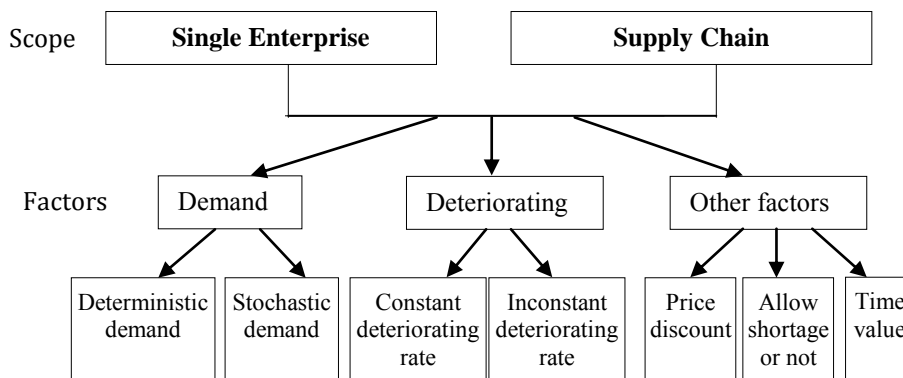


Fig. 1. Deteriorating item inventory review

Since 1975, studies have considered the effects of inflation on the inventory system. For the deterministic situation, Buzacott (1975) dealt with an economic order quantity model, with the inflation rate

subject to different types of pricing policies. Misra (1979) developed a discounted cost model and included the internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and time value of money on the order quantity with finite replenishment rate. Other studies considered variable demands, such as Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga and Ben-Daya (1996) and Chung (2003).

In most real-life situations, inflation is uncertain and unstable. Horowitz (2000) discussed an EOQ model with a normal distribution for the inflation rate. Furthermore, Mirzazadeh and Sarfaraz (1997) presented a multiple item inventory system with a budget constraint and a uniform distribution function for the external inflation rate. Additionally, Mirzazadeh presented the impact of uncertain inflationary conditions on inventory models using the average annual cost and the discounted cost (Mirzazadeh, 2007, pp. 658-666). In more recent research, Yang and Chang (2013) investigated a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Moreover, Lubik and Teo (2012) checked inventories, inflation dynamics and the New Keynesian Phillips curve. At the same time, Yang (2012) presented two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. Meanwhile, Neetu and Tomer (2012) presented a deteriorating inventory model under variable inflation where the supplier credits were linked to order quantity.

Previous studies of inventory systems have used the classical numerical method. In this paper, to minimize the discount cost method in the inflationary inventory model, we proposed a combined method. This involved Simpson's rules and the particle swarm optimization.

Particle swarm optimization (PSO) is a population-based stochastic optimization technique. It is based on the social behaviours observed in animals or insects, e.g., bird flocking, fish schooling and animal herding (Blum & Merkle, 2011). It was originally proposed by James Kennedy and Russell Eberhart (1995). In PSO, individual particles of a swarm represent potential solutions which move through the problem search space, seeking an optimal or satisfactory solution. The

particles broadcast their current positions to neighbouring particles. The position of each particle is adjusted according to its velocity (i.e., rate of change) and the difference between the best position found by its neighbours and the best position it has found so far. As the model is iterated, the swarm increasingly focuses on an area of the search space which contains high-quality solutions.

The following series of papers considered PSO for optimizing decision variables in the inventory control and supply chain management. Most of them used simple or hybrid methods with changed or combined parameters of PSO with other algorithms to find the answer: Sedighizadeh and Masehian (2009); Orand, Mirzazadeh, and Ahmadzadeh (2012); Parsopoulos, Skouri and Vrahatis (2008); Fatih Taşgetiren and Liang (2003); Tsai and Yeh (2008); Hsu, Tsou and Yu (2009); Chen and Su (2009); Domoto, Okuhara, Ueno and Ishii (2007); Varga, Király and Abonyi (2013); Goh Sue-Anna, S.G. Ponnambalama and N. Jawahar (2012), etc. Following these studies, the objective of this paper was to assess the performance of a method which solved the complex model with multi parameters. Hence, we combined PSO and Simpson's methods to optimize the discount cost value under the following assumptions.

The Assumption and Notations of the Model

Our development of the mathematical model of the discount cost was based on the following assumptions:

- The inflation rate is a random variable with normal distribution.
- The demand rate is known and constant.
- Shortages are allowed and fully backlogged.
- The replenishment is instantaneous and the replenishment cycle is the same for each period.
- Lead time is negligible and the initial inventory level is zero.
- The time horizon is infinite.
- A constant fraction of the on-hand inventory deteriorates per unit time, as soon as the item is received into inventory (Mirzazadeh, 2007, pp. 658-666).

The following notations were used in the mathematical model:

- Inflation rate per unit time

- Probability density function (pdf) of
- Mean value of
- Standard deviation of
- Interest rate per unit time
- Demand rate per unit time
- Constant deterioration rate, where $0 < \theta < 1$
- Inventory holding cost (for) and shortage cost (for) per unit per unit time at time zero
- Per unit purchase cost of the item at time zero
- Ordering cost per order at time zero
- Replenishment time interval
- Proportion of time in any given inventory cycle, which orders can be filled from the existing stock
- *EDC* Expected present value of costs
- Number of Particle
- Iteration
- Position particle in the iteration
- Velocity particle in the iteration
- Fitness particle in the iteration
- Best personal position of particle in the iteration
- Best group position of particles in the iteration
- Cognitive (individual) learning rate
- Social (group) learning rate
- Uniformly distributed random numbers in the range 0 and 1
- Coefficient of improvement PSO

The Discounted Cost Model

According to the inventory system costs, we considered EDCR, EDCP, EDCH and EDSC as the expected present value of the ordering, purchasing, carrying and shortage, respectively (for further elaboration, see reference Mirzazadeh, 2007, pp. 658-666). The expected present value of the replenishment cost of the (j+1) the cycle, was:

$$EDCR_j = E[A \cdot e^{-j(r-i)T}] \quad (1)$$

Therefore, the expected present value of the total replenishment cost was given by:

$$EDCR = E[A \cdot \sum_{j=0}^{\infty} e^{-j(r-i)T}] = A \cdot E\left[\frac{1}{1-e^{-(r-i)T}}\right] \quad (2)$$

We also assumed that the expected present value of the purchase cost for the first cycle was:

$$EDCP_1 = cE[D(e^{\theta kT} - 1)/\theta + DT(1-k) \cdot e^{-(r-i)T}] \quad (3)$$

Similarly for EDCR, the expected present value of the total purchase cost was:

$$EDCP = cE[(D(e^{\theta kT} - 1)/\theta + DT(1-k) \cdot e^{-(r-i)T}) \sum_{j=0}^{\infty} e^{-j(r-i)T}] \quad (4)$$

After simplifying, we had:

$$EDCP = \frac{cD}{\theta} (e^{\theta kT} - 1) E\left[\frac{1}{1-e^{-(r-i)T}}\right] + cDT(1-k) \left(E\left[\frac{1}{1-e^{-(r-i)T}}\right] - 1\right) \quad (5)$$

The expected present value of the holding cost of the first cycle was:

$$EDCH_1 = E\left[c_1 \int_0^{kT} I_1(t_1) e^{-(r-i)t_1} dt_1\right] = E\left[\frac{c_1 D[\theta(e^{-(r-i)kT} - 1) + (r-i)(e^{\theta kT} - 1)]}{\theta(r-i)(r-i+\theta)}\right] \quad (6)$$

Hence, the expected present value for the total holding cost was:

$$EDCH = E\left[\sum_{j=0}^{\infty} EDCH_1 e^{-j(r-i)T}\right] = E\left[\frac{c_1 D[\theta(e^{-(r-i)kT} - 1) + (r-i)(e^{\theta kT} - 1)]}{\theta(r-i)(r-i+\theta)(1-e^{-(r-i)T})}\right] \quad (7)$$

The expected present value of the shortages cost of the first cycle was:

$$EDCS_1 = E\left[-c_2 \int_0^{(1-k)T} I_2(t_2) e^{-(r-i)t_2} dt_2 e^{-(r-i)kT}\right] = E\left[\frac{c_2 D[(T(r-kr-i+ki)+1)e^{-(r-i)T} - e^{-(r-i)kT}]}{(r-i)^2}\right] \quad (8)$$

Thus, the expected present value for the total shortages cost was:

$$EDCS_1 = E\left[\sum_{j=0}^{\infty} EDCS_1 e^{-j(r-i)T}\right] = E\left[\frac{-c_2 D[(T(r-kr-i+ki)+1)e^{-(r-i)T} - e^{-(r-i)kT}]}{(r-i)^2(1-e^{-(r-i)T})}\right] \quad (9)$$

Consequently, the total expected discounted cost of the system was given by:

$$EDS(k, T) = EDCR(k, T) + EDCP(k, T) + EDCH(k, T) + EDCS(k, T) \quad (10)$$

EDC(k, T) is a multidimensional unconstrained optimization model

and we tried to optimize decision parameters k , T for obtaining an optimum ordering policy.

Particle Swarm Optimization

Particle swarm optimization (or PSO) is an evolutionary computational method. It is based on swarm intelligence and the behavior of a colony or swarm of insects such as ants or bees, a flock of birds or a school of fish (Rao, 2009, p. 725). This algorithm is based on the adaptive culture model and particle swarm which was first proposed by Kennedy and Eberhart in 1995. Kennedy theorized that the process of cultural adaptation comprises a high-level component. This is seen in the formation of patterns across individuals, the ability to solve problems, a low-level component, as well as the actual – and probably universal- behaviours of individuals. These can be summarized in terms of three principles (Kennedy *et al.*, 2001, p.287):

1. Evaluate
2. Compare
3. Imitate

According to these principles, training and learning do not occur unless these principles take place in society. Hence, Kennedy and Eberhart prepared a mathematical model of this algorithm which was developed based on the following model:

- When one bird locates a target or food (or maximum of the objective function), it instantaneously transmits the information to all other birds.
- All other birds gravitate to the target or food (or maximum of the objective function) but not directly.
- There is a component of each bird's own independent thinking, as well as its past memory (S. Rao, 2009, p.725; Kennedy, Eberhard & Shi, 2001, p.287).

a) The Algorithm

The following steps explain the method (S. Rao, 2009, p. 726):

- Assume the particle size N (number of particles).
- Generate an initial population of X in the range $X^{(l)}$ and $X^{(u)}$ randomly as X_1, X_2, \dots, X_N . (assume that i) and evaluate the

function.

- Find the velocities of particles. All particles will move to an optimal point at a certain velocity. The velocity is zero.

$$V_j^{(i)} = V_j^{(i-1)} + C_1 r_1 [P_{best,j} - X_j^{(i-1)}] + C_2 r_2 [G_{best} - X_j^{(i-1)}] \quad (11)$$

where C_1 and C_2 are the cognitive (individual) and social (group) learning rates, respectively, and r_1 and r_2 are uniformly distributed random numbers in the range 0 to 1. The parameters C_1 and C_2 denote the relative importance of the memory (position) of the particle itself to the memory (position) of the swarm. The values of C_1 and C_2 are usually assumed to be 1 so that $C_1 r_1$ and $C_2 r_2$ ensure that the particles would overfly the target about half the time. Furthermore, $P_{best,j}$ and G_{best} are the highest values of the objective functions for particle j and the highest values of the objective function $f[X_j^{(i)}]$ encountered in all previous iterations by any of the N particles, respectively (Rao, 2009, p.726).

- Find the position or coordinate of j^{th} particles in i^{th} iteration as:

$$X_j^{(i)} = X_j^{(i-1)} + V_j^{(i)} \quad j=1,2,N,N \quad (12)$$

where a time step of unity is assumed in the velocity term in Equation (12). Evaluate the objective function values corresponding to the particles as $f[X_1^{(i)}], f[X_2^{(i)}], \dots, f[X_N^{(i)}]$.

- Check the convergence of the algorithm if the algorithm has converged. Otherwise, stop updating iterations from to and start from Step 3.

b) Improvement PSO

Usually, the particle velocities build up too fast and the maximum of the objective function is skipped. Hence, an inertia term is added to reduce the velocity. The value of this is generally assumed to vary linearly from 0.9 to 0.4 as the iterative process progresses. The velocity of the particle, with the inertia term, is assumed as (Rao, 2009).

$$V_j^{(i)} = \theta V_j^{(i-1)} + C_1 r_1 [P_{best,j} - X_j^{(i-1)}] + C_2 r_2 [G_{best} - X_j^{(i-1)}] \quad (13)$$

The inertia weight θ was originally introduced by Shi and Eberhart in 1999. It compares to the original PSO algorithm in Equation (11). (Shi & Eberhart, 1998).

Simpson's Rules

In the numerical analysis, one method for numerical integration is Simpson's rule or Simpson's method. It divides the interval into smaller intervals, placing them in the integral formula. The smaller integrals are combined into a single unit to obtain the value of the function.

a) Composite Simpson's Rule

If the interval of integration is "small", then Simpson's rule will provide an adequate approximation of the exact integral. By small, what we really mean is that the function being integrated is relatively smooth over the interval. A smooth quadratic interpolate, like the one used in Simpson's rule, will give good results.

However, it is often the case that the function we are trying to integrate is not smooth over the interval. Typically, this means that either the function is highly oscillatory or it lacks derivatives at certain points. In such cases, Simpson's rule may give very poor results. One common way to handle this problem is to break the interval up into a number of small subintervals. Simpson's rule is then applied to each subinterval, with the results being summed to produce an approximation for the integral over the entire interval. This sort of approach is termed the composite Simpson's rule and is denoted by the following (Jeffrey & Dai, 1995, p.364; Wikipedia, 2012; Chapra, 2005, p.475):

$$\int_a^b f(x)dx = \frac{h}{3} \left[f(a) + f(b) + 2 \sum_{k=1}^{(n/2)-1} f(x_{2k}) + 4 \sum_{k=1}^{n/2} f(x_{2k-1}) \right] + R_n \quad (14)$$

where n is an even integer number, $h = (b-a)/n$, $x_k = a + h$ for $r = 0, 1, 2, \dots, n$, the error committed by the composite Simpson's rule is bounded (in absolute value) by:

$$R_n = -\frac{(b-a)h^4}{180} f^{(4)}(\xi) - \frac{nh^5}{180} f^{(4)}(\xi).$$

The Numerical Example

In this section, we provide two methods to solve this problem. The first is the classic numerical optimization method and the second is the composite Simpson's rules– PSO method. The first method only contains composite Simpson's rules and differential which you can see in the reference (Mirzazadeh, 2007). However, in the second method, we first use Simpson's method and afterwards, obtain two parameters that we want to be optimized. Following this, we use PSO to optimize the two parameters K and T . The values of the model parameters for the former method are as follows:

$D=1000$ units/year, $A=\$60$ /order, $c=\$1$ /unit, $c_1=\$0.2$ / unit/ year, $c_2=\$0.6$ / unit/ year, $r= \$0.2$ /\$/ year and $\theta=0.25$.

The inflation rate has the normal distribution function. The discount cost model that we showed with EDC (k,T) was an objective of this model. It was a multidimensional unconstrained optimization model. Hence, there were many optimization methods to solve the model. However, it is important to note that the points of this model have many parameters, with normal probability density function (pdf). Thus, derivative and integral makes an objective function more complex. For this reason, we faced constraints in choosing a solving method. We first tried a simple PSO to solve this model. After running the algorithm, we observed that the generated particles could not find a way to optimize point. This was because of complex integrals in the objective function and a lack of answer. Consequently, we combined PSO with Simpson approximation until the simplification function found the minimum of discount cost model. At first, we changed the function into Simpson's rules and obtained a function with two unknown decisions parameters, k and T . We then ran PSO. After doing this, the particles found their way to optimum point. For more details, see the following flow chart.

After running this application in C # we realized that the results of the Simpson method with $N = 4$ equaled the optimal values obtained in previous research (for further elaboration, see Mirzazadeh, 2007, pp.658-555). Thus, we changed the value of the N parameter in the Simpson's method. We observed that the function was reduced. This reduction continued until the value did not equal zero. This was

because in point of r the objectives function moved to infinity. Hence, in point of near $r = i$ the value of the objective function was very high and in other points, we observed the lower value of it. We initialized N , trying to obtain the best N for a good local minimum. Here, we investigated the values of N in Simpson's rules from $N = 1$ to 160. After obtaining a desired N value through the Simpson method, we investigated the convergence parameters of the PSO. For further elaboration, see Figure 3.

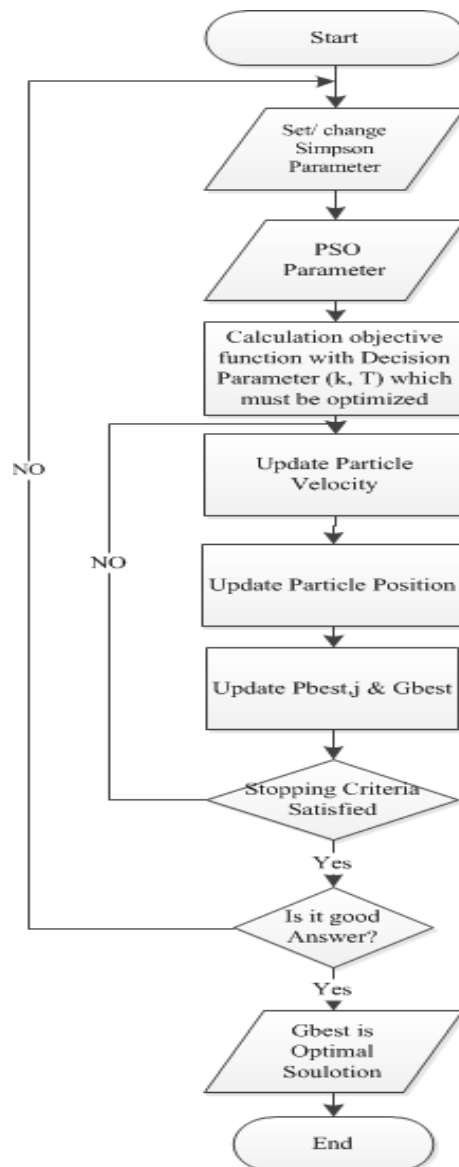


Fig. 2. Simpson's rules – particle swarm optimization

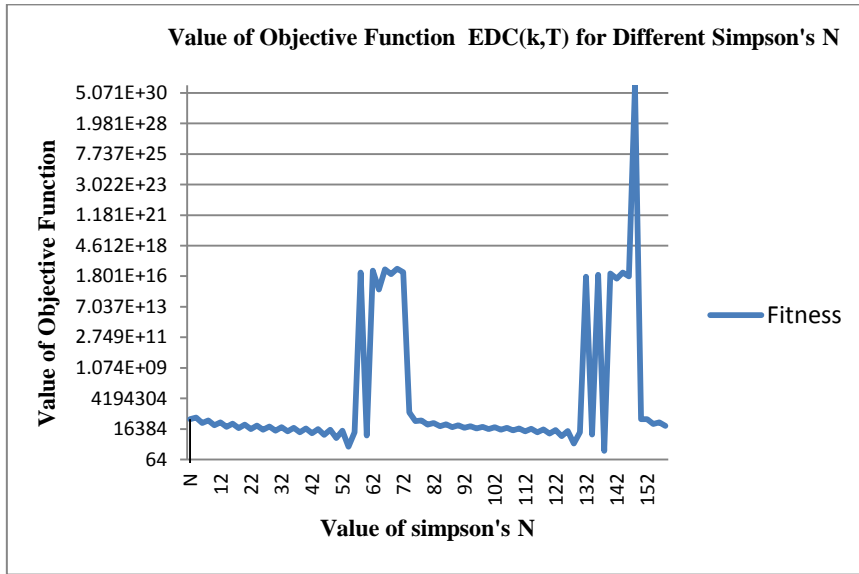


Fig. 3. Graphical representation of function EDC (k, T) with combination of Simpson and PSO for different N in Simpson's rules

The parameters of the algorithm were as follows: $N=30$, $i=100$, C_1 , $C_2=1$ and $r_1, r_2=$ random value in each iteration. The value of θ is usually assumed to vary linearly from 0.9 to 0.4 as the iterative process progresses. Following the algorithm parameters, we used $\theta = 0.4$ to reduce our velocity of particles to guarantee PSO convergence. The k and T values of the decision variables and fitness value of the function are shown in the following diagram:

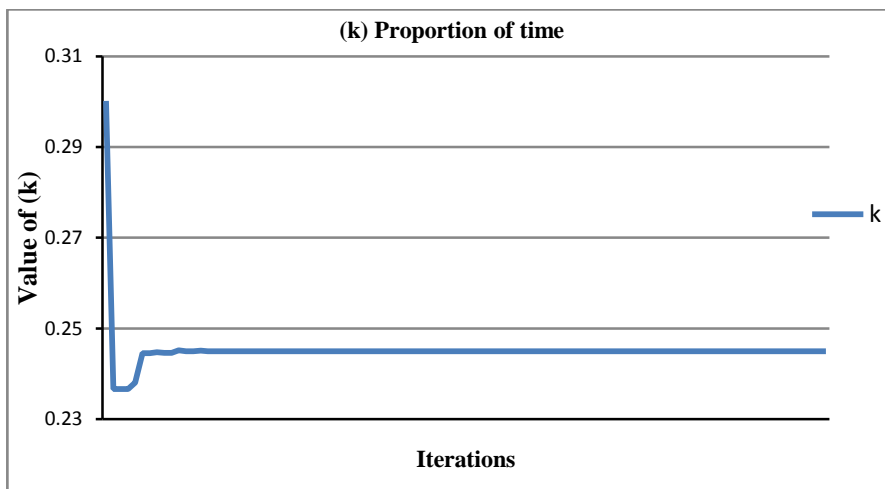


Fig. 4. Graphical representation convergence k parameter for Simpson N=52

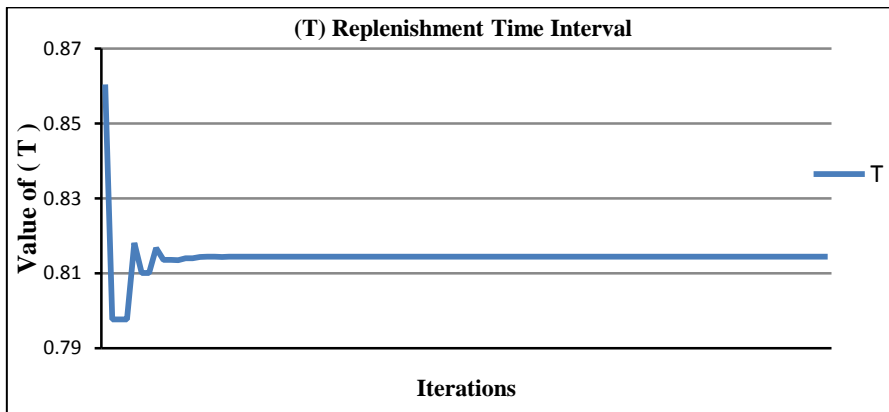


Fig. 5. Graphical representation convergence T parameter for Simpson N=52

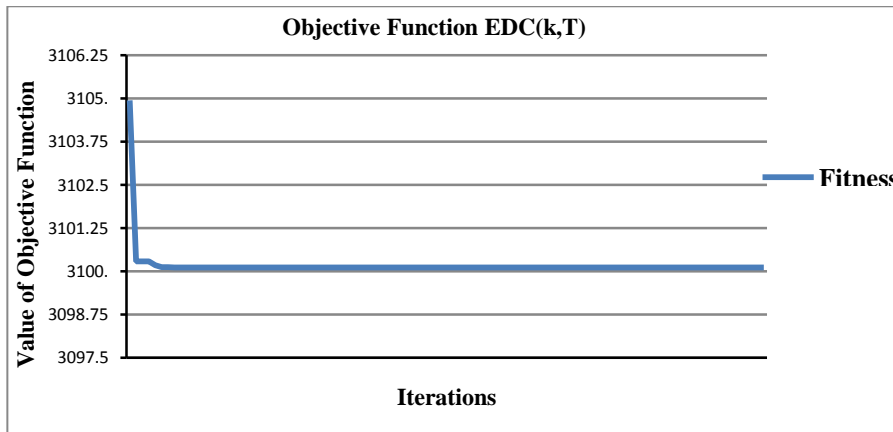


Fig. 6. Graphical representation convergence of function for Simpson N=52

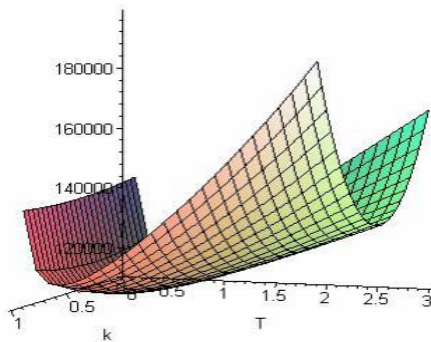


Fig. 7. Graphical representation of EDC (k, T) for $0 \leq T \leq 3$

The computational time of the algorithm was 100 milliseconds. The following table compares the results of this paper and those of previous studies.

Table 1 Optimal solution EDC (k, T)

Method	N Simpson	Time	k*	T*	EDC(k,T)*
Classical Method	4	4.46s	0.47843	0.67	98160.71
Particle Swarm Optimization	52	100ms	0.24500195	0.814401455	3100.112845

It is important to note that previous studies used Maple software, Simpson's rule and classical mathematical methods. In this paper, we used C# programming, PSO and a composite of Simpson's rule. However, in Maple, we first used Simpsons method. After obtaining two unknown parameters, we used differential to optimize k and T. Hence, due to the complex objective function, Maple did not seem to be a good program to complete this computation. In our method, we reduced the steps of calculation. Moreover, in equation condition, we obtained better optimized parameters. We prepared a parameter tuning on algorithm for more precision and checking computation time of the objective function for different PSO parameters. This is shown in Table 2.

Table 2. PSO parameter tuning of function for Simpson N=52

N	C1	C2	e	Iteration	EDC (k , T)	Time(Milliseconds)
20	0.8	0.8	0.9	100	3100.1128	76
25	0.85	0.85	0.8	150	3100.1128	132
30	0.9	0.9	0.7	200	3100.1128	202
35	1	1	0.6	250	3100.1128	292
45	1.4962	1.4962	0.7968	300	3100.1128	448

Conclusion

In this paper, we combined PSO and composite Simpson's rule for the optimization of a complex inventory model. The combination method helped the algorithm find a way to obtain decision parameter in models with complex integrals in PSO. In such models, particles cannot find the best way to reach an optimum point but the

combination method can improve it. Using classical methods yields the same results as using a combination method. Thus, we checked the time taken in each, as well as the number of steps. Our results showed that the combination method takes fewer steps and is faster than the old method. Given these findings, it seems clear that a combination method can help to optimize complex methods with mentioned specifications. Furthermore, it can reduce the number of steps and computation time of these models and help to find a better way to solve more complex inventory model with lower computation time in sensitivity analyses. Future research: 1. combine other algorithms with composite Simpson's rule, 2. combine other numerical method with Particle Swarm Optimization, 3. check the use of combined method for other complex formulae in the real world and other complex inventory control models.

References

- Kennedy, J. & Eberhart, R. (1995). *Particle swarm optimization* (Vol. 4). Piscataway, NJ: IEEE Press.
- Parsopoulos, K.; Skouri, K., & Vrahatis, M. (2008). *Particle Swarm Optimization for Tackling Continuous Review Inventory Models*. M. Giacobini et al. (Eds.): EvoWorkshops, 103–112.
- A.Lubik, T. & Teo, W. (2012). “Inventories, inflation dynamics and the New Keynesian”. *European Economic Review*. 56(3), 327–346.
- Blum, C., & Merkle, D. (2011). *Swarm Intelligence: Introduction and Applications*. Springer-Verlag.
- Buzacott, J. (1975). “Economic Order Quantities with Inflation”. *Journal of the Operational Research Society*, (26), 553-558.
- Chapra, S. (2005). *Applied Numerical Methods W/MATLAB: For Engineers & Scientists* (3rd ed., Vol. 1). New York, USA: McGraw-Hill Companies, Incorporated.
- Chung, K. (3003). “An Algorithm for an Inventory Model with Inventory-Level-Dependent Demand Rate”. *Computers and Operations Research*(30), 1311-1317.
- Datta, T., & Pal, A. (1991). “Effects of Inflation and Time-Value of Money on an Inventory Model with Linear Time-Dependent Demand Rate and Shortages”. *Journal of operational Research*, (52), 326-333.
- Domoto, E.; Okuhara, K.; Ueno, N. & Ishii, H. (2007). “Target Inventory Strategy in Multistage Supply Chain by Particle Swarm Optimization”. *Asia Pacific Management Review*, 12(2), 117-122.
- Fatih Taşgetiren, M. & Liang, Y.C. (2003). “A Binary Particle Swarm Optimization Algorithm for Lot Sizing Problem”. *Journal of Economic and Social Research*, 5(2), 1-20.
- Hariga, M., & Ben-Daya, M. (1996). “Optimal Time Varying Lot-Sizing Models under Inflationary Conditions”. *European Journal of operational Research*, (89), 313-325.
- Horowitz, I. (2000). “EOQ and Inflation Uncertainty”, *International Journal of Production Economics*, (65), 217-224.
- Hsu, C.H.; Tsou, C.S. & Yu, F.J. (2009). “Multicriteria Tradeoffs In Inventory Control Using Memetic Particle Swarm Optimization”. *International Journal of Innovative Computing, Information and Control*, 5(11), 3755–3768.
- Jeffrey, A. & Dai, H. H. (1995). *Handbook of Mathematical Formulas and Integrals* (4 ed., Vol. 1). (L. S. Yuhasz, Ed.) San Diego, California, USA: Academic Press.
- Kennedy, J.; Eberhart, R. & Shi, Y. (2001). *Swarm Intelligence* (Vol. 1). San Francisco, CA, USA: Morgan Kaufmann.

- Li, R.; Lan, H. & Mawhinney, J. (2010). "A Review on Deteriorating Inventory Study". *J. Service Science & Management*, 3, 117-129.
- Mirzazadeh, A. (2007). *Effects of Uncertain Inflationary Conditions on Inventory Models Using the Average Annual Cost and the Discounted Cost*. Bangkok: AIMS.
- Mirzazadeh, A. & Sarfaraz, A. (1997). *Constrained Multiple Items Optimal Order Policy under Stochastic Inflationary Conditions*. USA, San Diego.
- Misra, R. (1979). "A Note on Optimal Inventory Management under Inflation". *Naval Research Logistics Quarterly*, (26), 161-165.
- NEETU, & Tomer, A. (2012). *A Deteriorating Inventory Model Under Variable Inflation When Supplier Credits Linked to Order Quantity*. Procedia Engineering.
- Orand, S.; Mirzazadeh, A. & Ahmadzadeh, F. (2012). *Application of Particle Swarm Optimization Approach in the Inflationary Inventory Model under Stochastic Conditions*. Lulea.
- Rao, S. (2009). "Engineering optimization: Theory and practice". In *Engineering optimization: Theory and practice* (Vol. 1, pp. 708-714). Hoboken, New Jersey, USA: John Wiley and Sons.
- Sarker, B. & Pan, H. (1994). "Effects of Inflation and the Time Value of Money on Order Quantity and Allowable Shortage". *International Journal of Production Economics*, (34), 65-72.
- Sedighzadeh, D. & Masehian, E. (2009). "Particle swarm optimization methods, taxonomy and applications". *International Journal of Computer Theory and Engineering (IJCTE)*, 1(5), 486-502.
- Shi, Y. & Eberhart, R.C. (1998). *Parameter Selection in Particle Swarm Optimization*. Berlin: Springer-Verlag.
- Sue-Ann, G.; Ponnambalam, S. & Jawahar, N. (2012, October). "Evolutionary algorithms for optimal operating parameters of vendor managed inventory systems in a two-echelon supply chain". *Advances in Engineering Software*, 52, 47-54.
- Tsai, C.Y. & Yeh, S.W. (2008). "A multiple objective particle swarm optimization approach for inventory classification". *International Journal of Production Economics*, 656-666.
- Varga, T.; Király, A. & Abonyi, J. (2013). "Improvement of PSO Algorithm by Memory-Based Gradient Search—Application in Inventory Management". *Swarm Intelligence and Bio-inspired Computation Theory and Applications*. 403-422.
- Vrat, P. & Padmanabhan, G. (1990). "An Inventory Model under Inflation for Stock Dependent Consumption Rate Items". *Engineering Costs and Production Economics*, (19), 379-383.

- Wikipedia. (2012, December 28). *Simpson's Rule*. Retrieved from Wikipedia: http://en.wikipedia.org/wiki/Simpson's_rule.
- Wong, J.T.; Chen, K.H. & Su, C.T. (2009). "Replenishment Decision Support System Based on Modified Particle Swarm Optimization in a VMI Supply Chain". *International Journal of Industrial Engineering: Theory, Applications and Practice*, 16(1), 1-12.
- Yang, H.L. (2012). *Two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation*. *Int. J. Production Economics*.
- Yang, H.L. & Chang, C.T. (3013). "A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation". *Applied Mathematical Modelling*, 37(5), 2717–2726.