A Flexible Integrated Forward/ Reverse Logistics Model with Random Path-based Memetic Algorithm

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Abstract

Due to business and environmental issues, the efficient design of an integrated forward/reverse logistics network has recently attracted more attention from researchers. The significance of transportation cost and customer satisfaction spurs an interest in developing a flexible network design model with different delivery paths. This paper proposes a flexible mixed-integer programming model to deal with such issues. The model integrates the network design decisions in both forward and backward logistics networks, and also applies three kinds of delivering modes (normal delivery, direct shipment, and direct delivery) which enrich the model to be able to deliver the products to customers by distribution-skipping the mid-process strategy in order to deliver products in more flexible paths to customer zones. To tackle with such an NP-hard problem, a memetic algorithm (MA) with random path-based direct representation and combinatorial local search methods is developed. Numerical experiments are conducted to demonstrate the significance and applicability of the model as well as the efficiency and accuracy of the proposed solution approach.

Keywords

Integrated supply chain, Logistics network design, Random path-based direct encoding, Memetic algorithm.

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Introduction

Extended producer responsibility is becoming more common across the world. Implementation of legislation, social responsibility, corporate imaging, environmental concern, economic benefits and customer awareness are forcing original equipment manufacturers (OEMs) to manufacture products that are environmentally friendly in order to contribute to the global large-scale effort towards environmental protection. One way of doing so is through the utilization of returned products, which extends their useful life cycle.

Products can be returned for reasons such as customer dissatisfaction and warranty. Such products can be sorted for reuse, remanufacture, recycle and disposal. In addition, industries are using remanufacturing for expensive products such as turbines used in airplane and electricity generation systems. In these cases, the recovery of used products is economically more attractive than disposal (Koh, Hwang, Sohn, & Ko, 2002).

One of the most important and strategic issues in supply chain management is the configuration of the logistics network having a significant effect on the total performance of the supply chain. The configuration of the reverse logistics network, however, has a strong influence on the performance of the forward logistics network, and vice versa, as they share a number of resources. Due to the fact that designing the forward and reverse logistics separately leads to sub-optimal designs with respect to strategic and tactical costs, the design of the forward and reverse logistics networks should be integrated (Fleischmann, Beullens, Bloemhof, Ruwaard, & Wassenhove, 2001; Lee & Dong, 2008; Verstrepen, Crijnsen, de Brito, & Dullaert, 2007).

Previous research in the area of forward, reverse and integrated logistics network design was often limited to the consideration of the flow to be transported between two consecutive stages. In other words, there is no flow between facilities that are not consecutive. Nevertheless, considering flows between facilities that are not consecutive will enhance logistics network efficiency and flexibility.

Based on the aforementioned considerations, this paper addresses the issue of flexible, integrated, multi-stage forward/reverse logistics network design including suppliers, production, distribution,
collection/inspection, recovery and disposal facilities with limited capacity. The rest of this paper is structured as follows:

Section 2 offers a literature review to assess the state-of-the-art in forward/reverse logistics network design. To design flexible integrated forward/reverse logistics networks, a generalized mixed integer linear programming (MILP) formulation is developed in sections 3 and 4. Section 5 presents an efficient MA using a dynamic search strategy to find solutions for large-scale problems. The computational performance of the metaheuristic algorithm is analyzed in Section 6. Section 7 concludes this paper and offers guidelines for further research.

Literature review

This section presents a brief review of the most relevant and recent literature in closed-loop supply chain network design problems followed by two tables in order to categorize this area based on network types and demonstrate some gaps in this research field.

Integrated forward and reverse logistics refers to all those activities associated with the transformation and flows of goods and services with their information from the sources of the materials to the end users. Dullaert et al. (2007) gave a general review of the supply chain design models to support the development of richer supply chain models. These models range from simple uncapacitated facility location models to complex capacitated multi-objective models aimed at determining the cost minimizing or profit maximizing system design.

In the context of reverse logistics, various models have been developed in the last decade. Krikke et al. (1999) designed a MILP model for a two-stage reverse supply chain network for a copier manufacturer. In this model, processing costs of returned products and inventory costs are noticed in the objective function for minimizing the total cost. Fleischmann et al. (2001) designed a reverse logistics network by considering the forward flow together with the reverse flow, which has no capacity limitation. Extending Fleischmann et al.’s model, Salema et al. (2007) proposed a general model that has been applied to an Iberian company. However, when suspending the
logistics between dismantlers and plants, both Fleischmann et al. and Salema et al.’s models did not consider the supplier side, and lacked the relations between forward and reverse flows. Jayaraman et al. (1999) developed a MILP model for reverse logistics network design under a pull system based on customers’ demand for recovered products. The objective of the proposed model was to minimize total cost. Aras et al. (2008) developed a non-linear model and Tabu search solution approach for determining the locations of collection centers and the optimal purchase price of used products in a simple profit maximizing reverse logistics network. Biehl et al. (2007) simulated a carpet reverse logistics network, in which a specified experiment was used to analyze the effect of the system design and environmental factors influencing on the operational performance of the reverse logistics system. Kannan et al. (2010) developed a closed loop MILP model to determine raw materials, production, distribution and inventory, disposal, and recycling at different facilities. They presented a heuristic based on the genetic algorithm for their model minimizing the total supply chain costs. El-Sayed et al. (2010) presented a multi-period multi-echelon forward-reverse logistics network design model while the objective of their model was to maximize the profit of a supply chain. A bi-objective integrated forward/reverse supply chain design model was suggested by Pishvaee et al. (2010), in which the costs and the responsiveness of a logistics network were considered to be objectives of the model. They developed an efficient multi-objective priority-based MA by applying three different local searches in order to find the set of non-dominated solutions.

In addition, some researchers have presented studies on the optimization of stochastic supply chain network design in reverse logistics. Listeş & Dekker (2005) proposed a stochastic approach to the case study of recycling and demolition waste, while uncertainty was associated with demand source and quality. Pouralikhani et al. (2013) proposed an uncertain multi-period multi-stage closed-loop logistics network design model incorporating strategic network design decisions along with tactical material flow to avoid sub-optimality from separated design in both parts. The demands of first market customer zones are assumed to be stochastic. The problem is
formulated in a mixed integer non-linear programming (MINLP) decision making form as a multi-stage stochastic program with objective function maximizing the total expected profit.

In a practical case study of remanufacturing, Kerr et al. (2001) attempted to quantify the life cycle environmental benefits achieved by incorporating remanufacturing into a product system, based on a study of Xerox photocopiers in Australia. They found that remanufacturing could reduce resource consumption and wastes generation if a product is designed for disassembly and remanufacturing.

Most of previous researches have utilized mixed integer programming (MIP) to model the problem. These models range from simple single objective forward facility location models (Jayaraman & Pirkul, 2001) to complex multi-objective closed-loop models (Chaabane, Ramudhin, & Paquet, 2012).

Moreover, since the majority of logistics network design problems can be categorized as NP-hard, many powerful heuristics, metaheuristics and Lagrangian relaxation (LR)-based methods have been developed for solving these models. Dengiz et al., (1997) offered many examples of GA, demonstrating that it can be applied to a wide variety of applicative areas. In the reverse logistics, Min et al. (2006) also successfully used GA to develop a multi-echelon reverse logistics network for product returns.

Elhedhli & Merrick (2012) have considered emission costs alongside fixed and variable location and production costs in a forward SCND problem. They used a concave function to model the relationship between CO₂ emissions and vehicle weight. As the direct solution of their proposed model is not possible, Lagrangian relaxation is used to solve it. Wang et al. (2011) have considered the environmental concerns of forward SCND by proposing a multi-objective optimization model that captures the trade-off between the total costs and the environmental impacts. M. Pishvae, Torabi, & Razmi (2012) proposed a credibility-based fuzzy mathematical model for a forward supply chain network with three stages. Their model aims to minimize both the environmental impacts and total costs. They showed the applicability of the model as well as the usefulness of the solution method in an industrial case study.
Jandagh et al. (2011) discussed the applications of qualitative research methods in management sciences. The differences between quantitative and qualitative research were clarified, and the statistical methods suitable for such research were explored.

Safari et al. (2012) aimed at offering such a mathematical model where the coefficients and constants used have all been extracted based on the analysis of research and educational aspects of Shahed University. The proposed model was a lexicographic model with 36 decision variables that were broken down into two classes of university source variables (15) and university product variables. The model also included 49 goals, seven structural constraints and 20 integer variables.

Mansourfar (2013) attempted to evaluate the potential advantages of international portfolio diversification for East Asian international investors when investing in the Middle Eastern emerging markets. Overall, the results of both econometric and the metaheuristic optimization methods supported each other. The findings of that study highlight the potential role of the Middle Eastern equity markets in providing international portfolio diversification benefits for East Asian investors. It was also found that the long and the short-term efficient frontiers in any of the intra- or inter-regionally diversified portfolios did not provide similar benefits.

Hamid (2014) investigated whether there was any relationship between consumer attitude, perceived value, and green products. To establish such an assumption, a sample of 300 educated respondents was selected to participate in the survey. The study concluded that within the given context of a developing country, consumers had a negligible attitude to and low perceived value of green products. Hence, no significant relationship was found between attitude, perceived value, and green products.

Recently, the importance of collecting and treating end-of-life (EOL) products has increased substantially (Sasikumar et al., 2008a; Sasikumar et al., 2008b; Sasikumar et al., 2009; Pokharel et al., 2009) overviewed various aspects of reverse logistics (RL). Accordingly, the design of RL networks has interested researchers considerably (Kannan et al., 2012; Srivastava, 2008). Pishvaee, Kianfar, and Karimi (2010) developed a mixed integer linear
programming model for a multistage RL network in which both opening and transportation costs were taken into account. Since the model is NP-hard, they have proposed a simulated annealing algorithm with special neighbourhood search mechanisms. Kannan et al. (2012) considered the environmental impacts of the RL network model proposed by Pishvaea et al. (2010) by appending a carbon footprint term to the objective function. They tested their model with a case study in the plastic industry. Fonseca et al. (2010) developed a bi-objective model in which total costs and environmental impacts of an RL network are taken into account. By using two-stage stochastic programming, the uncertainty of both shipping costs and waste generation amount is captured in their model. They applied the model for a case in the province of Cordoba.

To avoid the sub-optimality that arises from the separate modelling of forward and reverse networks, many researchers have integrated forward and reverse network design, known as closed-loop SCND (CLSC) (Soleimani, Seyyed-Esfahani, & Kannan, 2014). Fleischmann et al. (2001) suggested a mixed integer linear programming (MILP) model for designing a CLSC. They showed that RL operations could often be efficiently integrated into existing forward logistics. Schultmann, Zumkeller, & Rentz (2006) considered EOL vehicle treatment in Germany. They concentrated on the flow of used products and reintegrated reverse flow of used products into their genuine supply chains. They have modelled RL with vehicle routing planning and finally have used Tabu search (TS) to solve the model.

Wang and Hsu (2010) have integrated environmental issues into an integer CLSC model, and have also developed a genetic algorithm (GA) based on a spanning tree structure to solve the propounded NP-hard model. Pishvaea and Razmi (2012) proposed a multi-objective fuzzy mathematical programming model for designing an environmental supply chain under inherent uncertainty of input data in such problem. They applied the life cycle assessment (LCA) method to quantify the environmental influence of the network.

Devika et al. (2014) simultaneously considered the three pillars of sustainability in the network design problem. A mixed integer programming model was developed for this multi-objective closed-loop supply chain network problem. In order to solve this NP-hard
problem, three novel hybrid metaheuristic methods were developed. Moreover, they have classified the published literature according to some main features. A more detailed classification of the literature is illustrated in Table 2 by considering three characteristics: modelling type, outputs of the models and solution approaches. To summarize Table 2, a coding system is presented in Table 1 by which the literature is reviewed in Table 2.

<table>
<thead>
<tr>
<th>Field</th>
<th>Title</th>
<th>Symbol</th>
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</thead>
<tbody>
<tr>
<td>Modeling</td>
<td>Stochastic mixed integer programming</td>
<td>SMIP</td>
</tr>
<tr>
<td></td>
<td>Fuzzy mixed integer programming</td>
<td>FMIP</td>
</tr>
<tr>
<td></td>
<td>Mixed integer non-linear programming</td>
<td>MINLP</td>
</tr>
<tr>
<td></td>
<td>Mixed integer linear programming</td>
<td>MILP</td>
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<tr>
<td>Solution</td>
<td>Exact</td>
<td></td>
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<tr>
<td>approach</td>
<td>Branch and bound</td>
<td>B&amp;B</td>
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<tr>
<td></td>
<td>Lagrangian relaxation-based</td>
<td>LR</td>
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<tr>
<td></td>
<td>Genetic algorithm</td>
<td>GA</td>
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<td></td>
<td>Simulated annealing</td>
<td>SA</td>
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<tr>
<td></td>
<td>Tabu search-based</td>
<td>TS</td>
</tr>
<tr>
<td></td>
<td>Interactive fuzzy solution approach</td>
<td>F</td>
</tr>
<tr>
<td>Outputs</td>
<td>Others heuristics</td>
<td>H</td>
</tr>
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Table 1. Coding of logistics network design research

<table>
<thead>
<tr>
<th>Field</th>
<th>Title</th>
<th>Symbol</th>
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<tbody>
<tr>
<td></td>
<td>Suppliers/orders</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>Facilities location</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>Facility capacity</td>
<td>FC</td>
</tr>
<tr>
<td></td>
<td>Allocation</td>
<td>Al</td>
</tr>
<tr>
<td></td>
<td>Production amount</td>
<td>PQ</td>
</tr>
<tr>
<td></td>
<td>Production assignment to production centers</td>
<td>PA</td>
</tr>
<tr>
<td></td>
<td>Utilization of production centers</td>
<td>UT</td>
</tr>
<tr>
<td></td>
<td>Transportation amount</td>
<td>TA</td>
</tr>
<tr>
<td></td>
<td>Transportation mode</td>
<td>TM</td>
</tr>
<tr>
<td></td>
<td>Delivery mode</td>
<td>DM</td>
</tr>
<tr>
<td></td>
<td>Inventory</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>Price of products</td>
<td>P</td>
</tr>
</tbody>
</table>
Many research directions still require intensive research in the area of closed-loop logistics network design problems. Moreover, since network design problems belong to the class of NP-hard problems, developing efficient solution methods is still a critical need in this area.

**Problem description**

As illustrated in Figure 1, in this paper an MILP model with seven echelons for a closed-loop supply chain network problem is proposed. The illustrated model emphasizes different delivery methods by considering three types of paths: i) normal delivery, in which products are delivered from one echelon to another adjoining one, ii) direct shipment delivery, in which products are delivered from plants to customers directly, and iii) direct delivery, in which products transport from DCs to customers or via plants to retailers directly.
The proposed model is based on the following common assumptions in the literature (Syarif, Yun, & Gen, 2002; H.-F. Wang & Hsu, 2010; Yao & Hsu, 2009):

- The demand of customers must be satisfied.
- The number of facilities in each echelon as well as their potential sites is restrained by pre-defined values.
- There is no flow between the facilities of the same echelon.
- The recovery and disposal rates are known in advance.
- In this supply chain network, there are a maximum of seven echelons: suppliers, plants, DCs, retailers, customers, collection/inspection centers, and disposal centers.
- Customers have no special preferences; that is, the price of products is the same wherever a customer buys them.

As a special characteristic of closed-loop logistics, suggested Van Der Laan et al. (1999), we assume that the number of the end-of-life products returned to the collection/inspection centers is a fraction of customers’ demands. In addition, they are allocated to the treatment facilities based on their qualities.

The problem in this study can be stated as follows:

**Given:**

- The set of potential sites for locating facilities;
- The set of available paths to deliver products to customers;
- The demand of customers;
• The cost of opening facilities and shipping materials;
• The fraction of end-of-life products in each customer zone that is returned to the respective collection/inspection center;
• The fraction of end-of-life products classified in the collection/inspection centers for each treatment center;
• The fraction of end-of-life products transported from each collection/inspection center to the plants;
• The capacity of each facility.

Determine:
• The configuration of supply chain network;
• The best route to deliver products to customer zones;
• The number of products that have to be manufactured at each plant;
• The assignment of customers to distribution centers and collection/inspection centers;
• The assignment of suppliers to plants.

The major contributions and features that distinguish this study from those previous are as follows:
- The forward and reverse logistics are integrated, while many previous supply chain networks are just forward-extended logistics networks (see Gen, Cheng, & Lin 2008);
- Different methods of delivering products to customers are considered in forward networks, while to the best of our knowledge there is no application of these methods in the closed-loop logistics network design (see Wang & Hsu, 2010; Devika et al. 2014);
- Applying the random path-based solution representation in the metaheuristic approach to tackle the NP-hard nature of the problem in closed-loop networks is the last contribution of this paper. It should be noted that previous works in this area have applied other solution representations such as priority-based encoding method (M.S. Pishvae, Farahani, et al., 2010), spanning tree-based (H.F. Wang & Hsu, 2010) or other approaches (see Govindan, Soleimani, & Kannan, 2014).

Model formulation
The notations of the proposed model are presented as below:
Indices

\begin{align*}
i & \quad \text{The Number of Suppliers with } i = 1, 2, \ldots, I \\
j & \quad \text{The Number of Plants with } j = 1, 2, \ldots, J \\
k & \quad \text{The Number of Distribution Centers (DCs) with } k = 1, 2, \ldots, K \\
l & \quad \text{The Number of Retailers with } l = 1, 2, \ldots, L \\
m & \quad \text{The Number of Customers with } m = 1, 2, \ldots, M \\
N & \quad \text{The Number of Collection/Inspection Centers with } n = 1, 2, \ldots, N \\
o & \quad \text{The Number of Disposal Centers with } o = 1, 2, \ldots, O 
\end{align*}

Parameters

\begin{align*}
cia_i & \quad \text{The Capacity of Supplier } i \\
cia_j & \quad \text{The Capacity of Plant } j \\
cia_k & \quad \text{The Capacity of Distribution Center } k \\
cia_l & \quad \text{The Capacity of Retailer } l \\
cia_m & \quad \text{The Demand of Customer } m \\
cia_n & \quad \text{The Capacity of Collection/Inspection Center } n \\
Ca_o & \quad \text{The Capacity of Disposal Center } o \\
p_r_m & \quad \text{The Recovery Percentage of Customer } m \\
pd_n & \quad \text{The Disposal Percentage of Collection/Inspection Center } n \\
ai & \quad \text{The Unit Cost of Transportation from Supplier } i \text{ to Plant } j \\
bjk & \quad \text{The Unit Cost of Transportation from Plant } j \text{ to DC } k \\
cjl & \quad \text{The Unit Cost of Transportation from Plant } j \text{ to Retailer } l \\
djm & \quad \text{The Unit Cost of Transportation from Plant } j \text{ to Customer } m \\
ekl & \quad \text{The Unit Cost of Transportation from DC } k \text{ to Retailer } l \\
fkm & \quad \text{The Unit Cost of Transportation from DC } k \text{ to Customer } m \\
glm & \quad \text{The Unit Cost of Transportation from Retailer } l \text{ to Customer } m \\
hnm & \quad \text{The Unit Cost of Transportation from Customer } m \text{ to Collection/Inspection Center } n \\
ilj & \quad \text{The Unit Cost of Transportation from Collection/Inspection Center } n \text{ to Plant } j \\
jno & \quad \text{The Unit Cost of Transportation from Collection/Inspection Center } n \text{ to Disposal Center } o \\
FCJ & \quad \text{Fixed Cost of Operating Plant } j \\
FCK_ & \quad \text{Fixed Cost of Operating DC } k 
\end{align*}
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$FCL_l$</td>
<td>Fixed Cost of Operating Retailer $l$</td>
</tr>
<tr>
<td>$FCN_n$</td>
<td>Fixed Cost of Operating Collection/Inspection Center $n$</td>
</tr>
<tr>
<td>$FCO_o$</td>
<td>Fixed Cost of Operating Disposal Center $o$</td>
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**Variables**

**Continuous Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XIJ_{ij}$</td>
<td>Quantity of Products Shipped from Supplier $i$ to Plant $j$</td>
</tr>
<tr>
<td>$XJK_{jk}$</td>
<td>Quantity of Products Shipped from Plant $j$ to DC $k$</td>
</tr>
<tr>
<td>$XJL_{jl}$</td>
<td>Quantity of Products Shipped from Plant $j$ to Retailer $l$</td>
</tr>
<tr>
<td>$XJM_{jm}$</td>
<td>Quantity of Products Shipped from Plant $j$ to Customer $m$</td>
</tr>
<tr>
<td>$XKL_{kl}$</td>
<td>Quantity of Products Shipped from DC $k$ to Retailer $l$</td>
</tr>
<tr>
<td>$XKM_{km}$</td>
<td>Quantity of Products Shipped from DC $k$ to Customer $m$</td>
</tr>
<tr>
<td>$XLM_{lm}$</td>
<td>Quantity of Products Shipped from Retailer $l$ to Customer $m$</td>
</tr>
<tr>
<td>$XMN_{mn}$</td>
<td>Quantity of Products Shipped from Customer $m$ to Collection/Inspection Center $n$</td>
</tr>
<tr>
<td>$XNJ_{nj}$</td>
<td>Quantity of Products Shipped from Collection/Inspection Center $n$ to Plant $j$</td>
</tr>
<tr>
<td>$XNO_{no}$</td>
<td>Quantity of Products Shipped from Collection/Inspection Center $n$ to Disposal Center $o$</td>
</tr>
</tbody>
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**Binary Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha_j$</td>
<td>If production takes place at Plant $j$</td>
</tr>
<tr>
<td>$\beta_k$</td>
<td>If a distribution center is opened at location $k$</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>If a distribution center is opened at location $l$</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>If a Collection/Inspection center is opened at location $n$</td>
</tr>
<tr>
<td>$\phi_o$</td>
<td>If a disposal center is opened at location $o$</td>
</tr>
</tbody>
</table>
Minimize \( Z = \sum_{i} \sum_{j} a_{ij} \cdot X_{IJ} + \sum_{j} \sum_{k} B_{jk} \cdot X_{JK} + \sum_{i} \sum_{l} c_{ij} \cdot X_{JL} + \sum_{j} \sum_{m} d_{jm} \cdot X_{JM} + \sum_{k} \sum_{m} e_{km} \cdot X_{KL} + \sum_{k} \sum_{m} f_{km} \cdot X_{KM} + \sum_{l} \sum_{m} g_{lm} \cdot X_{LM} + \sum_{m} \sum_{n} h_{mn} \cdot X_{MN} + \sum_{n} \sum_{j} l_{nj} \cdot X_{NJ} + \sum_{i} \sum_{n} j_{no} \cdot X_{NO} + \sum_{j} a_{j} \cdot F_{C} + \sum_{k} \delta_{i} \cdot F_{C} + \sum_{n} \lambda_{m} \cdot F_{C} + \sum_{o} \varphi_{o} \cdot F_{C} \)

The model seeks to minimize the total investment and operation costs of the flexible closed-loop logistics network design problem with products being simultaneously shipped and collected (1). These costs are depicted as i) transportation costs between facilities and depots as well as between depots and customers for each forwarded or collected product, and ii) investments required to open a facility.

The following constraints ensure that the flow of products is maintained and the demands are satisfied.

Subject to

\[ \sum_{i} X_{IJ} + \sum_{n} X_{NJ} = \sum_{j} X_{JK} + \sum_{l} X_{JL} + \sum_{m} X_{JM} \quad \forall j \quad (2) \]

\[ \sum_{j} X_{JK} = \sum_{k} X_{KL} + \sum_{m} X_{KM} \quad \forall k \quad (3) \]

\[ \sum_{k} X_{KL} = \sum_{m} X_{LM} \quad \forall l \quad (4) \]

\[ \sum_{j} X_{JM} = \sum_{k} X_{KM} + \sum_{l} X_{LM} = \quad \forall m \quad (5) \]

\[ \sum_{n} X_{MN} = pd_{n} \sum_{o} X_{NO} \quad \forall n \quad (6) \]

\[ \sum_{m} X_{MN} = \sum_{n} X_{NJ} \quad \forall n \quad (7) \]

The amount of products transmitted from each facility is restricted by its capacity in constraints 8 to 13:

\[ \sum_{j} X_{IJ} \leq c_{ai} \quad \forall i \quad (8) \]
The inherent characteristics of the Genetic Algorithm (GA) indicate why this algorithm may be a suitable method for flexible logistics network design problems. Applying a population of answers in many generations causes GA to search in multiple directions. On the other hand, lack of capacity of enough search intensification is often a disadvantage to a pure GA. To improve the intensification of the search in GA, Moscato & Norman (1992) first introduced Memetic Algorithm (MA). MA is a population-based metaheuristic algorithm that is similar to GA and has an additional local search to improve the intensification of GA toward global optimum answers. It has been proved that MA has practical success in a variety of problem domains such as NP-hard optimization models (Moscato & Cotta, 2003).

Applying domain knowledge and population-based search methods such as GA, as well as local search ones such as Simulated Annealing (SA), MA combine the advantages of both intensification and

\begin{align}
\sum_{k}^{XJK} jk + \sum_{l}^{XJL} jl + \sum_{m}^{XJM} jm & \leq \alpha_{j} \cdot caj_{j} \quad \forall j \quad (9)
\end{align}

\begin{align}
\sum_{l}^{XKL} kl + \sum_{m}^{XKM} km & \leq \beta_{k} \cdot cak_{k} \quad \forall k \quad (10)
\end{align}

\begin{align}
\sum_{l}^{XLM} lm & \leq \delta_{l} \cdot cal_{l} \quad \forall l \quad (11)
\end{align}

\begin{align}
\sum_{j}^{XNJ} nj + \sum_{o}^{XNO} no & \leq \lambda_{n} \cdot can_{n} \quad \forall n \quad (12)
\end{align}

\begin{align}
\sum_{o}^{XNO} no & \leq \varphi_{o} \cdot cao_{o} \quad \forall o \quad (13)
\end{align}

Constraint (14) ensures that the customer demand is satisfied:

\begin{align}
\sum_{j}^{XJM} jm + \sum_{l}^{XKM} km + \sum_{l}^{XLM} lm & \geq cam_{m} \quad \forall m \quad (14)
\end{align}

Finally, integrality and non-negativity of variables are guaranteed:

\begin{align}
\alpha_{j}, \beta_{k}, \delta_{l}, \lambda_{n}, \varphi_{o} & \in \{0, 1\} \quad \forall j, k, l, n, o \quad (15)
\end{align}

\begin{align}
XIJ_{ij}, XJK_{jk}, XJL_{jl}, XJM_{jm}, XKL_{kl}, XKM_{km}, XLM_{lm}, XMN_{mn}, XNJ_{nj}, XNO_{no} & \in N \cup \{0\} \quad \forall j, k, l, m, n, o \quad (16)
\end{align}

**Solution approach**

The inherent characteristics of the Genetic Algorithm (GA) indicate why this algorithm may be a suitable method for flexible logistics network design problems. Applying a population of answers in many generations causes GA to search in multiple directions. On the other hand, lack of capacity of enough search intensification is often a disadvantage to a pure GA. To improve the intensification of the search in GA, Moscato & Norman (1992) first introduced Memetic Algorithm (MA). MA is a population-based metaheuristic algorithm that is similar to GA and has an additional local search to improve the intensification of GA toward global optimum answers. It has been proved that MA has practical success in a variety of problem domains such as NP-hard optimization models (Moscato & Cotta, 2003).

Applying domain knowledge and population-based search methods such as GA, as well as local search ones such as Simulated Annealing (SA), MA combine the advantages of both intensification and
diversification of heuristic algorithms. Recently, MA has been applied in a variety of NP-hard problems such as production-distribution problems (Boudia & Prins, 2009), scheduling models (Tavakkoli-Moghaddam, Safaei, & Sassani, 2009), minimum span frequency assignment problems (Kim, Smith, & Lee, 2007), and partitioning problems (ElMekkawy & Liu, 2009). This paper applies an MA to the flexible closed-loop logistics model described as follows.

**Chromosome representation**

Although applying a flexible logistics model improves the flexibility and efficiency of the supply chain network, using the new delivery routes makes the problem much more complex. To tackle such an NP-hard problem, the random path-based direct encoding method is adopted; the ability of this approach to manage candidate solutions has been described by Gen & Cheng (1999). As shown in Figure 2, the chromosome’s length is $7 \times N$ (N is the number of customers), and every seven genes produces one unit, each of which denotes the delivery route to a customer as well as the recovery route from that customer.

![Figure 2. Chromosome representation for customer i](image)

The first three alleles of a unit represent the backward flow of the network, which shows the flow to collection/inspection centers, disposal centers, and plants, respectively. The four remaining alleles of that unit show the forward flow from suppliers to customers, where each of alleles represents the ID of retailers, DCs, plants, and suppliers, respectively. As an illustration, an ID randomly assigned to these facilities in the reverse and forward flow is shown in Figure 4. It
should be noted that applying this encoding approach might generate an unfeasible solution, which violates the facility capacity constraint. A repairing method is therefore imperative. As discussed earlier, in the delivery route to a customer at least one plant must be assigned. If the total demand of a customer from a plant exceeds its capacity, that customer will be assigned to another plant with sufficient product supply so that the transportation costs between that assigned plant and the customer is the lowest.

Inputs:
- M: The number of customers
- N: The number of collection/inspection centers
- O: The number of disposal centers
- L: The number of retailers
- K: The number of DCs
- J: The number of plants
- I: The number of suppliers

Begin
For \( i = 0 \) to \( N - 1 \)
- \( C_k[7i+1] \leftarrow \text{random}(1,N) \)
- \( C_k[7i+2] \leftarrow \text{random}(1,O) \)
- \( C_k[7i+3] \leftarrow \text{random}(1,J) \)
- \( C_k[7i+4] \leftarrow \text{random}(1,L) \)
- \( C_k[7i+5] \leftarrow \text{random}(1,K) \)
- \( C_k[7i+6] \leftarrow \text{random}(1,J) \)
- \( C_k[7i+7] \leftarrow \text{random}(1,I) \)
End
End

Output: The chromosome \( C_k \)

---

**Random path-based direct decoding**

The delivery and recovery paths can be conventionally determined by applying the random path-based direct decoding procedure. Using this method speeds up the algorithm. Table 3 and Figure 4 represent an instance of the delivery and recovery route in our model, which helps to understand better the decoding procedure for our approach.
### Table 3. The delivery and recovery path

<table>
<thead>
<tr>
<th>Delivery path</th>
<th>Forward Flow</th>
<th>Revers Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_2 - P_4 - DC_3 - R_6 - C_1 - CI_3 - (P_4, D_2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2 - P_4 - R_6 - C_1 - CI_3 - (P_4, D_2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2 - P_4 - DC_3 - CI_3 - (P_4, D_2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2 - P_4 - CI_3 - (P_4, D_2)$</td>
<td>3 2 4 6 3 4 2</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. A sample of gen

### Memetic operators

The role of operators in the quality of answers is unavoidable in MAs, and this paper has focused on the crossover and local search methods, population size, fitness function or selection approach and termination conditions. Inspired by the research conducted on these issues in the literature of metaheuristics algorithms in other fields of operations research, we applied the mostly compatible approaches to the closed-loop logistics network design problem with different delivery paths, as below:

**Cross over**

We applied the most common method for GAs and MAs: the two-point crossover. In this method we first generate two random positions: head and tail. Then, the substring between these positions is exchanged with the second chromosome in the same range.

**Local search**

After crossing, two offspring are created. The child with a better fitness function is selected for further improvement during the subsequent local search method in which a combinatorial crossover method has been applied, as explained below.

**Combinatorial local search**

In this approach, the most economic path in the neighbourhood of a customer is calculated, and the path with the lowest cost is selected. As illustrated in Figure 5 in a relatively narrow area, the trade-off between quality of the answers and consumption of CPU time is satisfactorily managed.
**Input:** problem data and qualified offspring from crossing

For $i = 1: M$

From all available routes from $C_k [ ]$

Calculate the total transportation cost through each path

Select the best path

**Output:** the best delivery path to a customer in the qualified offspring

**Fig. 5. Pseudo code of combinatorial local search**

### Selection

In the proposed MA, the popular roulette wheel selection approach is applied to select the next generation from the old population, making the selection probability of a chromosome proportional to its fitness value.

### Computational results

To test the accuracy and efficiency of the proposed MA, the following example is adopted as a base for comparison. To test the efficiency, different sizes of the test problems are produced through doubling the numbers of the nodes at each stage, as shown in Table 4, and running ten times for each problem. A total of 50 experiments were investigated by our algorithm. Other parameters are generated randomly using uniform distributions, as shown in Table 4. The results were compared with ILOG–CPLEX. These experiments were all performed on a PC with an Intel® Core™ i5 2.53 GHZ computer with 4 GB. Test problem 1 is the illustrative example of which $I = 1, J = 2, K = 5, L = 8, M = 20, N = 2, O = 1$. In test problem 2, $I = 2, J = 4, K = 10, L = 16, M = 40, N = 4$ and $O = 2$, there are 142 constraints, and 516 variables (including 36 binary variables), and the optimal solution is 50742 by LINGO. In test problem 3, $I = 4, J = 8, K = 20, L = 32, M = 80, N = 8$, and $O = 4$, there are 284 constraints, and 2012 variables (including 72 binary variables). The problem size increases to 568 constraints and 7848 variables in problem 4, and problem 5 reaches 1136 constraints and 30608 variables. We can observe that the size of the problem increased immensely.
To evaluate the efficiency and accuracy of our MA, LINGO was first implemented as it is the most commonly applied software. The results are illustrated in Table 6. For test problem 3, after $10^{12}$ iterations and 20 min(s) of elapsed runtime LINGO failed to obtain the final solution, as with test problem 4. For problem 5, LINGO failed to find a feasible solution before 40 min. With our MA, two cases were considered: one was done with the same population size of
80 for five test problems regardless of the problem size, and the other was done by increasing the population size with the problem size in order to obtain more accurate results. Table 6 summarizes the test results.

<table>
<thead>
<tr>
<th>Test problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>LINGO Optimal</td>
<td>50742</td>
<td>99120</td>
<td>195168</td>
<td>392931</td>
<td>789434</td>
</tr>
<tr>
<td>Time (s)</td>
<td>7</td>
<td>33</td>
<td>&gt;1800</td>
<td>&gt;1800</td>
<td>&gt;2400</td>
</tr>
<tr>
<td>MA Min_cost (pop-size=80)</td>
<td>50742</td>
<td>99220</td>
<td>196254</td>
<td>396258</td>
<td>798456</td>
</tr>
<tr>
<td>Ave_cost</td>
<td>50802</td>
<td>99248</td>
<td>205482</td>
<td>425648</td>
<td>851379</td>
</tr>
<tr>
<td>Ave_time</td>
<td>2.91</td>
<td>15.24</td>
<td>38.01</td>
<td>33.129</td>
<td>1508.84</td>
</tr>
<tr>
<td>MA Pop_size (variable pop-size)</td>
<td>50</td>
<td>80</td>
<td>200</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>Min_cost</td>
<td>50742</td>
<td>99220</td>
<td>195456</td>
<td>394576</td>
<td>788945</td>
</tr>
<tr>
<td>Ave_cost</td>
<td>51025</td>
<td>99248</td>
<td>204513</td>
<td>425111</td>
<td>819564</td>
</tr>
<tr>
<td>Ave_time</td>
<td>2.34</td>
<td>15.24</td>
<td>106.91</td>
<td>2150.68</td>
<td>10352.11</td>
</tr>
</tbody>
</table>

From Table 6 it can be observed that LINGO fails to solve such kinds of large-scale models, whereas our algorithm is capable of doing so. Besides, with our MA, increasing the population size with problem size only slightly improved accuracy of the problem, yet requires a large computation time. Therefore, we do not have to use a large population size to implement our algorithm as the problem size increases.

As far as LINGO cannot solve large-scale problems with its branch-and-bound method; CPLEX is considered to be more efficient by its branch-and-cut method, and thus was developed for more experiments. The comparisons of our algorithm and CPLEX with different error rates are illustrated in Table 7, of which the costs in boldface are the optimum answers.

\[
error\ rate = \frac{MA_{\text{answer}} - \text{CPLEX}_{\text{answer}}}{\text{CPLEX}_{\text{answer}}} \tag{17}
\]
Moreover, from Table 7, it can be illustrated that although a larger population size can improve the solution, it consumes huge computation time. The ‘trade-off’ between these is to find a suitable population size in the consideration of the error rate and CPU time. Therefore, if we set the acceptable error rate in advance, the respective population size can be determined. In our experiments, 7% of the acceptable error rate was assumed, and thus the population size of 80 was used. In reality, the control of error and making of effective decisions are the most important concerns of a company, which can then be achieved by setting a suitable population size under a required accuracy using our algorithm. In summary, the proposed random path-based memetic algorithm has demonstrated its performance in terms of both accuracy and efficiency.

### Conclusion and future remarks

In this paper, an MILP for a multi-stage closed loop supply chain network design problem is developed that aims to minimize the total cost of opening facilities and transportation in the proposed network. Many research studies have been conducted on closed-loop logistics network design problems in which the network includes suppliers, plants, DCs, customer zones, collection/inspection and disposal centers with normal delivery paths from two consecutive echelons of the network. However, there still exists a gap in both quantitative modeling and the solution approach of flexible multi-stage closed-

<table>
<thead>
<tr>
<th>Problem</th>
<th>ILOG-CPLEX</th>
<th>MA (variable pop_size)</th>
<th>Error rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min_cost</td>
<td>Ave_time(s)</td>
<td>Min_cost</td>
</tr>
<tr>
<td>1</td>
<td>50742</td>
<td>0.07</td>
<td>50742</td>
</tr>
<tr>
<td>2</td>
<td>99120</td>
<td>1.14</td>
<td>99220</td>
</tr>
<tr>
<td>3</td>
<td>195161 (feasible solution)</td>
<td>&gt;491.22 (Out of memory)</td>
<td>195456</td>
</tr>
<tr>
<td>4</td>
<td>391909 (feasible solution)</td>
<td>&gt;1165.34 (Out of memory)</td>
<td>394576</td>
</tr>
<tr>
<td>5</td>
<td>789152 (feasible solution)</td>
<td>&gt;1325.28 (Out of memory)</td>
<td>788945</td>
</tr>
</tbody>
</table>
loop logistics problems. The proposed model considering retailer centers in the network is able to build up a logistics network more productively and flexibly with different delivery modes.

In more detail, the proposed network has considered three kinds of delivering paths in its model that were not used in closed-loop network problems previously. These are: normal delivery, to transport products from an echelon to another adjoining one; direct shipment, to deliver products from plants to customers directly rather than via DCs and retailers; direct delivery, to transport products from DCs to customers, or from plants to retailers to customers directly. Applying this strategy in distribution, skipping the mid-process, leads to closer routes to customers, reduced transportation costs and lead times and increased customer satisfaction.

To solve the problem on larger scales a memetic algorithm with a combinatorial local search method is applied to find near optimum solutions in large-scale problems. Some test problems from small to large sizes are also solved with LINGO and CPLEX optimization software. Comparing the results obtained by exact methods via those obtained by the memetic algorithm proved the algorithm’s accuracy, capability, and efficiency.

The proposed future remarks are as below:
- Other objective functions, such as responsiveness, tardiness, and robustness can be considered as other goals in designing the flexible logistics network design problem.
- The uncertainty embedded in demand and recovery rates can be examined in a more analytical way to make the model closer to reality.
- The proposed random path-based direct encoding approach and combinatorial local search method for the flexible closed-loop logistics network design problem can be applied in other metaheuristic algorithms such as cloud theory-based simulated.
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