

## **Estimating Stock Price in Energy Market Including Oil, Gas, and Coal: The Comparison of Linear and Non-Linear Two-State Markov Regime Switching Models**

**Reza Mohseni\*, Leila Sakhtkar Modallal**

*Faculty of Economic and Political Sciences, Shahid Beheshti University, Tehran, Iran*

(Received: December 24, 2016; Revised: May 16, 2017; Accepted: May 30, 2017)

### **Abstract**

A common method to study the dynamic behavior of macroeconomic variables is using linear time series models; however, they are unable to explain nonlinear behavior of the series. Given the dependency between stock market and derivatives, the behavior of the underlying asset price can be modeled using Markov switching process properties and the economic regime significance. In this paper, a two-state Markov switching model in energy market has been examined for oil, coal, and gas since 1991 to 2011. The objective price estimated by the switching model and the parameters were determined by using MATLAB program. With regard to the relationship between the total price and the variables defined in this paper, it is concluded that the non-linear model is relatively better than the linear model, since it has lower RMSE and greater R-squared, therefore it is better to use nonlinear model in Markov switching model for predicting the price of stocks.

### **Keywords**

Markov switching regime models, linear and non-linear two-state Markov switching models, Energy market.

---

\* Corresponding Author, Email: [re\\_mohseni@Sbu.ac.ir](mailto:re_mohseni@Sbu.ac.ir)

## **Introduction**

In view of the fact that the value of derivatives is derived from the underlying asset price, considering the fluctuations as constant, Black and Scholes (1973) have explained the dynamics of the underlying asset based on the Brownian process.

In recent years, there has been an increasing interest in the use of regime-switching models to capture the macroeconomic regimes affecting the behavior of the market increase of crude oil prices (1973-1974), the stock market crash (1987), the financial crisis in East Asia such as China, Taiwan, Singapore, South Korea in 1997, and the financial crisis of (2007-2008), also known as the global financial crisis, are the examples of those events which led to changes in the processes of financial time series, stock market shock and oil price shock are the main driving forces behind the credit default market and stock market All of these countries caught up in the Asian financial crisis of 1997 show trade deficits. The reason of the transformation from commercial surplus to fraction for these countries was related to China's decision of entering into export-led growth. While China had less educated workers and a larger internal market in comparison with Southeast Asia, it quickly drew exports from the countries of Southeast Asia. Generally, two categories of internal (environmental) and external factors are cited for the crisis of the East Asian countries. The most important internal factors are the exchange rate stabilization, liberalization of capital movements, trade account deficit, weakening competition power, and reducing exports of the Southeast Asian nations, and the most important external factors are the celerity of regional rate of economic activity in Asia, the crisis spread to all regional countries and devaluation of the Chinese currency (Gill et al., 2016).

The capital asset pricing model (CAPM) is a model that describes the relationship between systematic risk and expected return for assets, particularly stocks. CAPM is widely used throughout finance to estimate risky securities, producing expected returns for assets considering the risk of those assets and appraising capital costs. This is an idealized representation of how financial markets price

securities, and thereby determining the expected returns on capital investments. This model provides a methodology for quantifying risk and translating it into estimates of expected return on equity. Since it necessarily simplifies the global financial markets, CAPM cannot be used in isolation. Everyone wants to make a profit by trading on the market, and in this context a so called portfolio is a dream come true; this is one of the central concepts of this theory.

Regime switching is a model which could explain and measure these changes. Regime switching models can effectively calculate the intricate attributes of time series including interest and exchange rates. Using regime switching models, researches have shown the development of econometrics and have proven the superiority of this model in comparison with other models (Cheng-Der et al., 2012). Market interest rate, the drift and the volatility of the underlying risky asset switch over time according to the state of an economy, which is modelled by a continuous-time Markov chain. Regime switching models seek to capture discrete shifts in the behavior of financial variables; this action will be done by allowing the parameters of the underlying data generating process take on different values in different time periods. And also evidence shows that spillover effects between several kinds of markets have widely been discussed around the globe whereas the transmission of shocks between oil market and stock returns has received little attention.

Oil price shocks have nonlinear effects on stock returns. Empirical evidence from international stock indexes suggests that an increase in oil prices has a negative and significant impact on stock prices in one state of the economy, whereas this effect is significantly dampened in another state of the economy.

The remainder of this paper is organized as follows. Section 2 explains the pertaining literature. Section 3 describes the methodology of the research and introduces the model. The result is then described in Section 4 and Section 5 concludes the paper with a summary.

## **Literature Review**

Most of the empirical studies have concentrated on the structural

shortcomings of the underlying asset market in recent years. The rise and fall of oil prices during above-mentioned period (1991-2011) such as significant increases from 2003 to 2008 and decreases from 2008 to 2009, stock market crash, and some other variations led to changes in the stages of financial time series. Accordingly, the regime switching model was widely used in stock market and derivatives. The main idea is considering a change in regime behavior.

Out of sample forecast performance of two parametric and two non-parametric nonlinear models of stock returns done by Kanas (2003). In his work, the parametric models include the standard regime switching and the Markov regime switching, whereas the non-parametric are the nearest-neighbor and the artificial neural network models. Ultimately, it was shown that Markov regime switching model is the most preferable non-linear extension of the present model for stock return forecasting.

Hamilton (1994) studied a regime switching model with similar steps in each regime, and established a logarithmic likelihood function based on the possibility of regimes for parameters estimation, and then maximized it. Using a generalized regime switching model, Mun Fung and Hock See (2002) consider the temporary state of volatility of daily returns on crude oil futures which lead to sudden changes in mean and variance, GARCH dynamics, basis-driven time-varying transition probabilities and conditional leptokurtosis. This flexible model provides many compound features of conditional volatility within a relatively economical configuration. They conclude that regime switching models prepare a favorable framework for the financial historians who are interested in studying factors beyond the progress of volatility and oil futures traders engrossed in short-term volatility forecasts. Nomikos and Alizade (2004), by using Markov Regime Switching (MRS) models, describe a new approach in order to determine time-varying minimum variance hedge ratio in stock index futures markets. The rationale behind the use of these models stems from the fact that the dynamic relationship between spot and futures returns may be characterized by regime shifts, which, in turn, suggests that by allowing the hedge ratio to be dependent upon the state of the

market, more efficient hedge ratios may be obtained and therefore superior hedging performance may be compared to other methods in the literature. Over all, the results indicate that market agents may be able to increase the performance of their hedges which are measured in terms of variance reduction and increase in their utility by using MRS models.

Nomikos and Alizade (2004) examined the efficiency of hedge proportions generated from Markov regime switching models in the oil futures markets. Their study was based on the dynamic relevance between spot and futures prices that may be determined by regime changes. They also present a Markov regime switching vector error correction model with GARCH error structure. Their results show that hedging effectiveness is significantly improved in most cases by allowing the variances and covariance of spot-futures returns and also the error correction coefficients to be state dependent.

Zou and Chen (2013) use two-state Markov-switching model to explain behavior of crude oil for 26 years, and found that composite likelihood approach can better show changes in oil prices. Balcilar et al. (2015) investigated the relationship between US crude oil and stock market prices by using a Markov-switching model and monthly data since 1859 to 2013. According to the variance-covariance matrix of the oil and stock prices, they approximated the two regime models divided into high and low volatility regimes and realized that the high-volatility regime existed before the Great Depression and after the oil price shock in 1973. The low-volatility regime often arises when the oil markets drop the control of the international oil companies. Additionally, they found that the high-volatility regime presumably happens in the time of recession. In order to analyze the sudden changes' transmission in volatility conduction from the energy market across several energy indices including Romania, (Acatrinei, 2015) applied a MSGARCH model. The category of Markov Switching GARCH (MSGARCH), as well as GARCH models, may provide a declaration of changes in the conditional volatility. They use daily data for ten years in order to take the dependencies and sensitivities of energy. The results show the volatility level changes and regime

duration shift among countries resulted from the heterogeneity of the firms included in the energy equity indices, because they are subject to manifold risk factors. The results do not show any emerging clear pattern in the long term, but once abrupt and potent shocks happened in the short term; there are overlapping patterns in the high volatility regime. Accordingly, the energy indices have responded simultaneously with the decrease in the oil prices in December 2014 denoting an evident dependency and sensitivity to oil price. Energy indices in the European Union had an equivalent pattern with the emerging markets and countries which have fast-growing economies such as Brazil, Russia, India, and China in December 2014.

Mamipour and Vaezi Jezai (2015) use a cointegrated vector autoregressive Markov-switching model to examine the nonlinear properties of three variables in energy market for ten years and found that impact of oil price on stock returns is positive and significant in the short run and negative in the long run.

### Methodology

Markov chain is a stochastic process where the transition from one state to other is possible and countable. Its memorylessness, that is the conditional probability distribution of next state depends only on the current state and is not related to earlier events.

$$P(X(t_{n+1}) = j | X(t_1) = i_1, \dots, X(t_n) = i) = P(X(t_{n+1}) = j | X(t_n) = i) \quad (1)$$

where

$$P(X(t_1) = i_1, \dots, X(t_n) = i) > 0 \quad (2)$$

This relation is a Markov property. Transition (probability) function is,

$$P(s, i, t, j) := P(X(t) = j | X(s) = i) \quad \forall 0 \leq s \leq t, i, j \in s \quad (3)$$

Also  $P(s, I, t, j)$  represents probability of transition process to state  $j$  in time  $t$  from state  $i$  in time  $s$  (Everitt & Skrondal, 2010).

**Underlying Asset and the Markov Regime-Switching Model**

We consider a financial market with  $M$  economic situations. Given the volatility and movement, the underlying asset can adopt different values in different regimes. To describe and model this market, a continuous-time Markov chain  $\varepsilon_t$  with  $M$  status was considered, so the underlying asset model is as follows:

$$\frac{dS_t}{S_t} = \mu_{\varepsilon_t} dt + \sigma_{\varepsilon_t} dW_t \quad 0 < t < T \tag{4}$$

where  $dW_t$  is a Wiener process (the process of geometric Brownian motion),  $\sigma_{\varepsilon_t}$  is the underlying asset volatility (stock),  $\mu_{\varepsilon_t}$  is a risk-free interest rate movement, and  $S_t$  is the underlying asset.  $\sigma_{\varepsilon_t}$  and  $\mu_{\varepsilon_t}$  depend on the Markov chain  $\varepsilon_t$  which can adopt different values in different regimes. The model above is a Markov regime-switching model (Karatzas & Shreve, 1991).

In the two-state Markov we have:

$$\begin{aligned} \varepsilon_t = 0 & \text{ when function cycle is expansionary} \\ \varepsilon_t = 1 & \text{ when function cycle is contraction} \end{aligned} \tag{5}$$

In expansion mode,  $\varepsilon_t = 0$ ,  $\mu_{\varepsilon_t} = \mu_0$ , and  $\sigma_{\varepsilon_t} = \sigma_0$ , and in contraction mode  $\varepsilon_t = 1$ ,  $\mu_{\varepsilon_t} = \mu_1$ , and  $\sigma_{\varepsilon_t} = \sigma_1$ ; we suppose that  $\sigma_0 \neq \sigma_1$ .

In addition to three-state hidden Markov models we have three different states: Expansion, transition, contraction. We consider economic growth as the beginning of expansion. In this case we assume (Lawrence & Rabiner, 1980):

$$\begin{aligned} \varepsilon_t = 2, \mu_{\varepsilon_t} = \mu_2 \text{ and } \sigma_{\varepsilon_t} = \sigma_2 \\ \varepsilon_t = 1, \mu_{\varepsilon_t} = \mu_1 \text{ and } \sigma_{\varepsilon_t} = \sigma_1 \\ \varepsilon_t = 0, \mu_{\varepsilon_t} = \mu_0 \text{ and } \sigma_{\varepsilon_t} = \sigma_0 \end{aligned} \tag{6}$$

$\varepsilon_t = 2$ ,  $\varepsilon_t = 1$  and  $\varepsilon_t = 0$ , respectively represents expansion, transition and contraction states. So in three states Markov  $\varepsilon_t$  is as follows:

$$\begin{aligned} \varepsilon_t = 2 & \text{ when function cycle is expansionary} \\ \varepsilon_t = 1 & \text{ when function cycle is transition} \\ \varepsilon_t = 0 & \text{ when function cycle is contraction} \end{aligned} \tag{7}$$

We suppose that  $P_k = P_k(S_k, t)$  is the price of an option with  $k$

regimes  $k = 1, 2, \dots, M$ . Economic situations were determined through continuous time Markov chain,  $\varepsilon_t = \{\varepsilon_t, t \geq 0\}$ .

### The Two-State Markov Switching Model in Energy Market

The purpose of this survey is utilizing the two-state Markov switching model in the energy market of oil, coal, and gas since 1991 to 2011 in Iran. In this part, we estimate the price and determine its parameters. With regard to the relationship between the total price and variables defined in this study (oil, coal, and gas), the proposed model is expressed in linear and non-linear forms in such a way that the effect of the defined variables is obvious.

In the Markov regime-switching models the regime is the unobservable variable. Generally, the two-state Markov switching model is as follows:

$$P_{i,t} = \begin{cases} \beta_{1,t}^1 X_{1,t} + \dots + \beta_{k,t}^1 X_{k,t} + \beta_{k+1,t} X_{k+1,t} + \dots + \beta_{j,t} X_{j,t} + \varepsilon_{t,1} & S_t = 1 \\ \beta_{2,t}^2 X_{1,t} + \dots + \beta_{k,t}^2 X_{k,t} + \beta_{k+1,t} X_{k+1,t} + \dots + \beta_{j,t} X_{j,t} + \varepsilon_{t,2} & S_t = 2 \end{cases} \quad (8)$$

In the above model,  $P_t$  is the dependent variable,  $X$  is the independent variable and  $S$  represents the number of states.

### The linear two-state Markov switching model

In the linear model, the parameter  $P_{i,t}$  is the total objective function and the total price parameter. The desired parameters in this example are  $X_1$  to  $X_3$ .

$$P_{i,t} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \quad (9)$$

where

$X_1$ : The price of relative value of stock index for Oil

$X_2$ : The price of relative value of stock index for Gas

$X_3$ : The price of relative value of stock index for Coal

$\beta_0$  to  $\beta_3$  are coefficients of the model,  $\beta_0$  is fixed coefficient,  $\beta_1$  is oil coefficient,  $\beta_2$  is gas coefficient, and  $\beta_3$  is coal coefficient.

The outcome of MATLAB application is the form of output coefficients and the best model for the desired function would be obtained after the insertion of these coefficients.

### The non-linear two-state Markov switching model

This model is as follow:

$$P_{i,t} = \beta_0 + \beta_1 X_1^{a_1} + \beta_2 X_2^{a_2} + \beta_3 X_3^{a_3} \quad (10)$$

$a_1$  to  $a_3$  are exponential factors, and the other parameters have been defined in the previous section. The data are derived from securities and the stock exchange organizations and they are related to the monthly relative values of each row. In this case, the aim is to obtain the optimal value of  $\beta$ , so that the objective function (total price)  $P_{i,t}$  is optimized and its value in comparison with the observed value (relative total price of reported indices) has the lowest error. The data are derived from the average daily data and, to synchronize data in terms of global price, each unit is presented in the form of stock exchange indices. We used MATLAB program to model this example.

So that all data are normalized using the following formula:

$$X = 0.1 + 0.9 \times ((X - X_{min}) / X_{max} - X_{min}) \quad (11)$$

The aim of normalizing data is to make them similar in terms of the numerical value. The entire collection would be ambiguous if data are placed at zero intervals, so after normalizing we will not see any data as zero or one.

### Results

We consider oil, gas and coal prices; all three variables have been driven from Iran Stock Exchange. These prices are derived for a period of 20 years (1991 to 2011), so that the data are obtained from average daily prices and they are monthly relative value of each item. In order to unify them in terms of global prices, each module is provided in the form of stock exchange indices.

### Calculation of the Linear Model

First, the relative prices of the three cases mentioned are normalized and then by writing a command line in MATLAB with circulation period of 500 and  $1 \times 1 - 10^{-16}$  error, linear coefficients of the model are obtained.

$$P_{i,t} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \quad (12)$$

**Table 1. Linear parameters estimation (Parameters obtained from MATLAB software)**

Coefficient	Estimation
Fixed $\beta$	0.0065
Oil $\beta$	1.6306
Gas $\beta$	0.0853
Coal $\beta$	0.0188

Now we set these parameters into the objective function:

$$P_{i,t} = 0.0065 + 1.6306X_1 + 0.0853X_2 + 0.0188X_3 \quad (13)$$

### Calculation of the Non-Linear Model

Considering the exponential coefficients and a little change in linear model, nonlinear coefficients of model are obtained.

$$P_{i,t} = \beta_0 + \beta_1 X_1^{a_1} + \beta_2 X_2^{a_2} + \beta_3 X_3^{a_3} \quad (14)$$

**Table 2. Non-linear parameters estimation (Parameters obtained from MATLAB software)**

Coefficient	Estimation
Fixed $\beta$	0.30140
Oil $\beta$	0.6221
Gas $\beta$	0.3371
Coal $\beta$	0.6110
$a_1$	0.4474
$a_2$	0.4625
$a_3$	0.7070

Now to evaluate and determine how well the model fits the data, we use the most common goodness of fit,  $R^2$  and then  $RMSE$  to verify compliance.

$R^2$  and  $RMSE$  for the linear model:

$$R^2 = 0.119 \quad (15)$$

$$RMSE = 0.264$$

$R^2$  and  $RMSE$  for the non-linear model:

$$R^2 = 0.24 \quad (16)$$

$$RMSE = 0.186$$

In general, the fraction of the variance of the dependent variable in a regression is represented by r-squared. In this way, values of r-squared closer to one are thought to be indicative of a *better* regression.

RMSE has also been used for further investigation. It is the standard deviation of the residuals (prediction errors). Residuals are a measure of how far from the regression line data points are (Brooks, 2014).

Therefore, by analyzing the result of the Markov switching model for the linear and non-linear models to estimate prices, coefficients of these two models were determined. As we see R Squared ( $R^2$ ) for the non-linear model is more than the linear one and also Root Mean Square Error (RMSE) for the non-linear model is less. So the non-linear model is more accurate than the linear model. In these models, the objective function is to optimize the sum of squares of the differences between prices for oil, gas, and coal and the predicted values. There are many ways for price estimation, no one manner is necessarily better or worse than the other, actually, their strengths and weaknesses are mostly complimentary to each other, for example, Wang et al. (2016) employed a Markov switching multifractal volatility model to forecast crude oil return volatility in stock market. He proves that MSM models can make more precise volatility forecasts. They are even better than GARCH-class models or the historical volatility models. They compute 1 and 20-day-ahead forecasts of MSM, GARCH-class and historical volatility models. In order to examine the fact that whether the differences between the loss functions of the different models are statistically significant, they also use an advanced test of the model confidence set. That is, MSM models have greater forecasting abilities rather than the GARCH or HV models. Examining the temporal behavior of volatility of daily returns on crude oil futures, Mun Fung demonstrates that the regime switching model performs perceptibly better than non-switching models irrespective of evaluation criteria.

According to this survey, in order to predict the price of stocks, it is better to use nonlinear model in Markov switching model.

The presence of important discrete economic events induces substantial non-linearity in the stochastic process and distorts inference in the event that it is not appropriately modeled. These procedures may lead to biased or inefficient results at any rate when regime shifts are deterministic instead of stochastic. In such cases, a nonlinear Markov-switching model proposed by Hamilton (1994) represents a lasting alternative to allow for stochastic behavioral changes.

Markov methodological limitations include such rules which have the highest priority and apply before all. Its application in systems with too much rules is difficult, so the efficiency will be slower. Most non-linear techniques give good in-sample fit data. A time-varying process has estimation difficulties when a shift in parameters occurs and the degradation of performance of structural macroeconomic models seems at least somewhat owing to regime shifts so that linear technics are not capable of describing these shifts. Markov regime switching model represents a very popular class which includes some alternative nonlinear and time-varying models. Generally, the model generates conditional heteroskedasticity and non-normality; prediction intervals are asymmetric and reflect the prevailing uncertainty about the regime.

### **Conclusion**

In this paper, we surveyed the two-state Markov switching model in energy market including oil, gas, and coal, and found that the Markov switching regime model is superior to Black-Scholes for the analysis of empirical phenomena in stock market. The main reason for the study of this model is the replacement of perfect and precise markets by asymmetry specification (skewness or smiles effect) related to fluctuations in market. Therefore, using this model in critical situations would automatically switch the market. So the proposed model for explaining various behavioral patterns in different time intervals is appropriate. Whereas Markov regime switching models can be considered as generalization of Hidden Markov models, the method proposed in this paper with two state regimes would compose

---

a more naturalistic model. The data are average of monthly prices of three items and the time period of the analysis extends from 1991 to 2011. We discussed the usage of the two-state Markov switching model by analyzing the results of the Markov switching model for the linear and non-linear models in energy market (oil, gas and coal). The result is that the non-linear model is relatively better than the linear model, because it has lower RMSE and greater R-squared.

This review may be useful for stakeholders, speculators and policy makers who are interested in the time-varying and volatile structure of the energy market.

Given the importance of the financial markets, some offers for future studies are using Markov switching models under random fluctuations such as the random variation of Hull-White, Heston, and others, study of the three-state regime or more and further review of diffusion and distribution test in order to achieve the appropriate model.

## References

- Acatrinei, M. (2015). Volatility spillover across energy indices of the stock markets. *Romanian statistical review*, 63(Issue 2), 5-13.
- Balcilar, M., Gupta, R., & Miller, S. M. (2015). Regime switching model of US crude oil and stock market prices: 1859 to 2013. *Journal of Energy Economics*, 49(C), 317-327.
- Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, 3(81), 637-654.
- Brooks, C. (2014). *Introductory Econometrics for Finance* (3<sup>rd</sup> ed.). England: Cambridge University Press.
- Cheng-Der, F., Ho, K. W. R., Hu, I., & Wang, R. H. (2012). Option pricing in a Black-Scholes model with Markov switching. *Journal of Data Science*, 10, 483-509.
- Everitt, B. S., & Skrondal, A. (2010). *The Cambridge Dictionary of Statistics* (4<sup>th</sup> ed.). Cambridge: Cambridge University Press.
- Gill, B., Goh, E., & Huang, C. (2016). *The dynamics of US-China-Southeast Asia relations*. Sydney: The University of Sydney.
- Hamilton, J. D. (1994). *Time series Analysis*. Princeton University Press.
- Kanas, A. (2003). Non-linear forecasts of stock returns. *International Journal of Forecasting*, 22(4), 299-315.
- Karatzas, I., & Shreve, S. E. (1991). *Brownian motion and Stochastic Calculus* (2nd ed.). New York, NY: Springer-Verlag.
- Lawrence, R. R. (1989). A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition. *Proceedings of the IEEE* 77, (2), 257-286.
- Mamipour, S., & Vaezi Jezai, F. (2015). Non-linear relationships among the oil price, the gold price and stock market returns in Iran: A multivariate regime-switching approach. *Iranian Journal of Economic Studies*, 4(1), 101-128.
- Mun Fung, W., & Hock See, K. (2002). A Markov switching model of the conditional volatility of crude oil futures prices. *Journal of Energy Economics*, 24(2000), 71-95.
- Nomikos, N., & Alizade, A. (2004). A Markov regime switching approach for hedging stock indices. *Futures Markets*, 24(7), 649-674.
- Wang, Y., Wu, C., & Li, Y. (2016). Forecasting crude oil market volatility: A Markov switching multifractal volatility approach. *International Journal of Forecasting*, 32, 1-9.
- Zou, W., & Chen, J. (2013). A Markov regime-switching model for crude-oil markets: Comparison of composite likelihood and full likelihood. *Canadian Journal of Statistics*, 41, 353-367.