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Relation between Imprecise DESA and MOLP Methods

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Abstract

It is generally accepted that Data Envelopment Analysis (DEA) is a method for indicating efficiency. The DEA method has many applications in the field of calculating the relative efficiency of Decision Making Units (DMU) in explicit input-output environments. Regarding imprecise data, several definitions of efficiency can be found. The aim of our work is showing an equivalence relation between one of the models of DEA with imprecise data and Multiple Objective Linear Programming (MOLP). The relation between DEA and MOLP was studied to use interactive multiple objective models for solving the DEA problem in exact situation and find the most preferred solution. The aim of this study is to analyze an equivalent relation between imprecise DEA (IDEA) and MOLP models. In this context, we tried to solve IDEA models with interactive project procedure. The Project method is the responsible method, because it can estimate any efficient solution, and it indicates Most Preferred Solution (MPS). In addition, we will use the Data Envelopment Scenario Analysis (DESA) model. The main characteristic of DESA model is to decrease all inputs and increase all outputs and estimate one problem instead of n problems.

Keywords

Data envelopment analysis, imprecise data, interactive multiple objective linear programming, project algorithm.

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Introduction

In various planning problems, it is necessary to obtain a solution that optimize all the objective functions at the same time. The last ten years have seen a huge growth in Multi-Objective Linear Programming (MOLP). Data envelopment Analysis (DEA) is defined by charnes et al. (1978) and MOLP are useful for management to draw a scheme for future. DEA is used to evaluate the past preferences, while MOLP is used for future preferences. Connection between DEA and MOLP have received much attention in the past decade. Ebrahimnejad and Tavana (2014) proposed an interactive MOLP procedure for describing target unit in DEA. Tavana et al. (2017) considered a hybrid DEA-MOLP for public school assessment. It would be worth mentioning that the important result obtained by this equivalence relation is using interactive MOLP to solve DEA model and obtain Most Preferred Solution (MPS). Imprecise DEA (IDEA) is defined by Cooper et al. (1999) to mean that data were vague, which results in a non-linear DEA model and we can transform it into a linear problem. Also, we obtain a double of interval DEA models that will be utilized for interval data rather than for crisp data.

The aim of this study is to analyze the connection between MOLP and DEA models in exact environment, and expand this relation between MOLP and DEA models with imprecise data. Following that, we use the interactive MOLP method with the name of *Project method* (Luque, 2009) to solve the IDEA. The suggested Project algorithm is a kind of interactive local trade-off method. It is based on the projection of utility function gradients onto the tangent hyperplane of an efficient set and on a local search technique that inherits the profits of the reference-point method to search for the best solution within a local area. Therefore, we can obtain every efficient solution.

On the other hand, previous works have more focused on certain conditions. More recent evidences (Yang & Xu, 2014) proved that the minimax approach is similar to a kind of DEA model. Thereafter Luque (2015) proposed a set of new reference points called ‘‘equivalent reference points’’ which builds the same solution in the reference point-based approaches (minimax approach). In addition, we would find the MPS in the IDEA models by the interactive algorithm that uses reference points, like Project method.

Data Envelopment Scenario Analysis (DESA)

We chose data envelopment scenario analysis (DESA) model particularly due to being the best procedure for this study. DESA model was defined by Thanassoulis and Dyson (1992) to discuss the uselessness of radial projection in target setting. In this model, the assumption is that $I=\{1,...,m\}$ showing that I is a set of inputs and $O=\{1,...,s\}$ showing that O is a set of outputs. By substituting $I \cong I_g \cup \bar{I}_g$ and $O \cong O_g \cup \overline{O}_g$, where O_g and I_g are used to show outputs and inputs where borders of success are used in the target model. The DESA problem can be expressed as follows:

$$
\begin{aligned}
\max_{\lambda_j, Z_k, \theta_i} \quad & \sum_{k\in O} P_k^+ Z_k - \sum_{i\in I} P_i^- \theta_i \\
st \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = \theta_i x_{io} \qquad i \in I \\
& \sum_{j=1}^{n} \lambda_j y_{kj} = Z_k y_{ko} \qquad k \in O \\
& \theta_i x_{io} \ge G_i \qquad i \in I_g \\
& Z_k y_{ko} \le G'_k \qquad k \in O_g \\
& A_i \le \theta_i \le 1/B_i, \qquad A_i, B_i \in [0,1], \qquad \forall i \in I \\
& \Gamma_k \le 1/Z_k \le 1/\Delta_k, \qquad \Gamma_k, \Delta_k \in [0,1], \qquad \forall k \in O \\
& \lambda_j \ge 0 \qquad \forall j
\end{aligned}
\tag{1}
$$

P_i^- and P_k^+ are the decision makers' preferences for the recovery of inputs and outputs. θ_i is the contraction rate of input *i* and Z_k is the development rate of output *k*. G_i and G'_k are borders for *i*th input and *k*th output, respectively. (A_i, B_i) and (Γ_k, Δ_k) are remarked as the width and height borders for θ_i and Z_k, respectively.

DESA Method with Imprecise Data

The aim of this study is to analyze and modify the DESA model with imprecise data, such that the aim of this change is to calculate one model instead of *n* models, this method can be formulated as follows:

$$
\begin{aligned}
&\underset{\lambda_j, Z_k, \theta_i}{Max} \quad \sum_{k\in O} P_k^+ Z_k - \sum_{i\in I} P_i^- \theta_i - M\sum_{i\in I_f} s_i' - M\sum_{k\in O_f} s_k'' \\
&st \quad \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}\, x_{ij} = \theta_i \sum_{j=1}^{n} x_{ij} \qquad i\in I \\
&\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} = Z_k \sum_{r=1}^{n} y_{kr} \qquad k\in O \\
&\sum_{j=1}^{n} \theta_i x_{ij} \ge G_i^L - s_i' \qquad i\in I_f \\
&\sum_{r=1}^{n} Z_k y_{kr} \le G_k'^U + s_k'' \qquad k\in O_f \\
&\sum_{j=1}^{n} \lambda_{jr} = 1 \qquad \forall r \\
&\lambda_j \ge 0 \quad \forall j, \quad \theta_i \; free, \quad Z_k \; free, \quad k\in O \\
&s_i' \ge 0 \quad \forall i\in I_f, \quad s_k'' \ge 0 \quad \forall k\in O_f
\end{aligned}
\qquad (2)
$$

$\left[G_i^L, G_i^U\right]$ and $\left[G_k'^L, G_k'^U\right]$ are the interval resources for all input consumption and all output generation, respectively. s_i', s_k'' are intended for the legalization of all diminution and all generation, respectively. *M* shows a penalty factor that has to be intended by the decision maker.

Imprecise DESA Model Based on Interval Arithmetic

In this section, the authors use Wang's model (2005) for transforming the above model to two new models with the names of undesirable model and desirable model. With this in mind, the imprecise DESA problem can be outlined in terms of best lower bound and best upper bound for each DMU, where $x_{ij} \in \left[x_{ij}^L, x_{ij}^U\right]$, $y_{kj} \in \left[y_{kj}^L, y_{kj}^U\right]$, $\theta_i \in \left[\theta_i^L, \theta_i^U\right]$, $Z_k \in \left[Z_k^L, Z_k^U\right]$.

$$
\begin{aligned}
\underset{\lambda_j, Z_k, \theta_i}{Max} \quad & \sum_{k\in O} P_k^+ Z_k^L - \sum_{i\in I} P_i^- \theta_i^L - M \sum_{i\in I_f} s_i' - M \sum_{k\in O_f} s_k'' \\
st \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}\, x_{ij}^L = \theta_i^L \sum_{j=1}^{n} x_{ij}^U \qquad i\in I \\
& \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}\, y_{kj}^U = Z_k^L \sum_{r=1}^{n} y_{kr}^L \qquad k\in O \\
& \sum_{j=1}^{n} \theta_i^L x_{ij}^U \ge G_i^U - s_i' \qquad i\in I_f \\
& \sum_{r=1}^{n} Z_k^L y_{kr}^L \le G_k'^L + s_k'' \qquad k\in O_f \\
& \sum_{j=1}^{n} \lambda_{jr} = 1 \qquad \forall r \\
& \lambda_{jr} \ge 0 \quad \forall j, \quad \theta_i^L\ free, \quad Z_k^L\ free, \quad k\in O \\
& s_i' \ge 0 \quad \forall i\in I_f, \quad s_k'' \ge 0 \ \ \forall k\in O_f
\end{aligned} \tag{3}
$$

G_i^U is the limit to all use of input x_{ij}^U, and $G_K'^L$ is the limit to total production of output y_{kr}^L

$$
\begin{aligned}
\underset{\lambda_j, Z_k, \theta_i}{Max} \quad & \sum_{k\in O} P_k^+ Z_k^U - \sum_{i\in I} P_i^- \theta_i^U - M \sum_{i\in I_f} s_i' - M \sum_{k\in O_f} s_k'' \\
st \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \gamma_{jr}\, x_{ij}^L = \theta_i^U \sum_{j=1}^{n} x_{ij}^L \qquad i\in I \\
& \sum_{r=1}^{n}\sum_{j=1}^{n} \gamma_{jr}\, y_{kj}^U = Z_k^U \sum_{r=1}^{n} y_{kr}^U \qquad k\in O \\
& \sum_{j=1}^{n} \theta_i^U x_{ij}^L \ge G_i^L - s_i' \qquad i\in I_f \\
& \sum_{r=1}^{n} Z_k^U y_{kr}^U \le G_k'^U + s_k'' \qquad k\in O_f \\
& \sum_{j=1}^{n} \lambda_{jr} = 1 \qquad \forall r \\
& \gamma_{jr} \ge 0 \quad \forall j, \quad \theta_i^U\ free, \quad Z_k^U\ free, \quad k\in O \\
& s_i' \ge 0 \quad \forall i\in I_f, \quad s_k'' \ge 0 \ \ \forall k\in O_f
\end{aligned} \tag{4}
$$

Where G_i^L is the limit to total consumption of input x_{ij}^L and $G_K'^U$ is the limit to total production of output y_{kr}^U.

With regard to the upper part, we can say Model (3) shows lower bound of the best feasible efficiency, and Model (4) demonstrates the upper bound of the best possible relative efficiency.

Establish an Equivalence Relationship Between DESA with Imprecise Data and MOLP

In this section, we want to introduce a multi-objective linear program (MOLP) and show that this model and the imprecise model are equivalent. In desirable model, suppose that:

$$\bar{f}_k(\lambda) = \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{U} \qquad k = 1...s \tag{5}$$

$$\tilde{f}_i(\lambda) = \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}x_{ij}^{L}, \qquad i = 1..m \tag{6}$$

$$\lambda = (\lambda_{11}, \lambda_{jr}, ..., \lambda_{nn})^T$$

It is assumed that $f_k' = \bar{f}_k(\lambda^*)$ is the maximum possible rate for all the kth outputs and $f_i'' = \tilde{f}_i(\lambda^*)$ is the minimum possible rate for all the ith inputs. Where λ^* can be computed by finding the lower part of the model:

$$\begin{aligned} &Max \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}^{U} \\ &s.t \quad \lambda \in \Omega \end{aligned} \tag{7}$$

$$\begin{aligned} &Min \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}x_{ij}^{L} \\ &s.t \quad \lambda \in \Omega \end{aligned} \tag{8}$$

Such that $f' = [f_1', ..., f_s']$ and $f'' = [f_1'', ..., f_m'']$ indicate the desirable points of Model (5) and Model (6). Suppose that:

$$\sum_{j=1}^{n}x_{ij} > 0 \quad i = 1,..,m \qquad \sum_{r=1}^{n}y_{kr} > 0 \quad k = 1,..,s$$

$$w_k = \frac{1}{\sum_{r=1}^{n}y_{kr}^{U}} \quad \forall k \qquad v_i = \frac{1}{\sum_{j=1}^{n}x_{ij}^{L}} \quad \forall i \tag{9}$$

And

$$\bar{f}_k^* = \frac{F^{\max}}{w_k} = F^{\max}\sum_{r=1}^{n}y_{kr}^{U}, \quad \tilde{f}_i^* = \frac{H^{\min}}{v_i} = H^{\min}\sum_{j=1}^{n}x_{ij}^{L} \tag{10}$$

$\tau_i = \theta_i^U - H^{\min}$, $\gamma_k = F^{\max} - Z_k^U$

Such that

$$F^{\max} = \max_{1\le k\le s}\{w_k f_k'\} = \max_{1\le k\le s}\left\{\frac{f_k'}{\sum_{r=1}^{n} y_{kr}^U}\right\} \quad (11)$$

$$H^{\min} = \min_{1\le i\le m}\{v_i f_i''\} = \min_{1\le i\le m}\left\{\frac{f_i''}{\sum_{j=1}^{n} x_{ij}^L}\right\} \quad (12)$$

We can write the equivalence model as the following model:

$$\begin{aligned}
&\underset{\lambda_j, Z_k, \theta_i}{Max} && \sum_{k\in O} P_k^+ Z_k^U - \sum_{i\in I} P_i^- \theta_i^U \\
&st && \tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 \qquad i = 1,\dots,m \\
& && Z_k^U \frac{1}{w_k} - \bar{f}_k(\lambda) = 0 \qquad k = 1,\dots,s \qquad (13)\\
& && g_i v_i \le \theta_i^U \qquad i \in I_f \\
& && g_k' w_k \ge Z_k^U \qquad k \in O_f \,, \quad \lambda \in \Omega
\end{aligned}$$

Also, we can write (14)

$$\tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 \Leftrightarrow v_i \tilde{f}_i(\lambda) = \theta_i^U \Leftrightarrow v_i \tilde{f}_i(\lambda) - H^{\min} = \theta_i^U - H^{\min}$$

$$\Leftrightarrow v_i(\tilde{f}_i(\lambda) - \tilde{f}_i^* = \theta_i^U - H^{\min}$$

Therefore (15)

$$Z_k^U \frac{1}{w_k} \bar{f}_k(\lambda) = 0 \Leftrightarrow -w_k \bar{f}_k(\lambda) = -Z_k^U \Leftrightarrow F^{\max} - w_k \bar{f}_k(\lambda) = F^{\max} - Z_k^U$$

$$\Leftrightarrow w_k(\bar{f}_k^* - \bar{f}_k(\lambda) = F^{\max} - Z_k^U$$

Such that $P_k^+ = P_i^- = 1, k \in O, i \in I$

Accordingly, above changes lead to the following model:

$$\underset{\lambda_j, Z_k, \theta_i}{Max}(\sum_{k\in O} Z_k^U - \sum_{i\in I}\theta_i^U) = \underset{\lambda_j Z_k, \theta_i}{Min}(\sum_{i\in I}\theta_i^U + \sum_{k\in O}(-Z_k^U)) \Rightarrow$$

$$Min(\sum_{i\in I}(\theta_i^U - H^{\min}) + \sum_{k\in O}(F^{\max} - Z_k^U) = Min\ (\sum_{i\in I}\tau_i + \sum_{k\in O}\gamma_k)$$

Also, for any $\lambda \in \Omega$, we have:

$$F^{\max} - Z_k^U \geq w_k \bar{f}_k^* - Z_k^U \geq w_k \bar{f}_k(\lambda) - Z_k^U = 0 \qquad k = 1,\dots,s$$

$$\theta_i^U - H^{\min} \geq \theta_i^U - v_i \tilde{f}_i^* \geq \theta_i^U - v_i(\tilde{f}_i(\lambda) = 0 \qquad i = 1,\dots,m$$

$$\bar{f}_k^* = \frac{F^{\max}}{w_k} \geq \frac{w_k f_k'}{w_k} = f_k' = \max_{\lambda\in\Omega} \bar{f}_k(\lambda) \qquad k = 1,\dots,s$$

$$f_i^* = \frac{H^{\min}}{v_i} \leq \frac{v_i f_i''}{v_i} = f_i'' = \min_{\lambda\in\Omega} \tilde{f}_i(\lambda) \qquad i = 1,\dots,m$$

In the end, we can rewrite the imprecise DESA model as a minimax formulation as follows:

$$\begin{aligned}
Min \quad & \sum_{i\in I}\tau_i - \sum_{k\in O}\gamma_k \\
st \quad & v_i(\tilde{f}_i(\lambda) - \tilde{f}_i^*) = \tau_i \qquad i = I \\
& w_k(\bar{f}_k^* - \bar{f}_k(\lambda)) = \gamma_k \qquad k = O \\
& g_i v_i - H^{\min} \leq \tau_i \qquad i \in I_f \\
& g_k' w_k - M \leq \gamma_k \qquad k \in O_f
\end{aligned} \tag{16}$$

$$\sum_{j=1}^{n}\lambda_{jr} = 1, \qquad \forall r \quad \lambda_{jr} \geq 0, \qquad M \geq 0 \qquad j, r = 1,\dots,n$$

Therefore, imprecise DESA model and minimax MOLP formulation are equivalent, this result leads to using interactive methods for solving imprecise DESA models and obtaining the most preferred solution.

Project Algorithm for Solving DESA Model with Imprecise Data

Project method is a comprehensive interactive method, this method is described according to the Grist method and reference point method (Luque, 2009). In the literature, Grist method tends to be used to define a normal vector on the efficient frontier. The Project method is introduced as one of the reference point methods. This method proposed a new search for obtaining the best answer. Project algorithm is defined as follows:

Step 1: In the initial stage of the process, we can obtain the best value for both inputs and outputs,

And denoted by $f'_k = \bar{f}_k(\lambda^*)$, $f''_i = \tilde{f}_i(\lambda^*)$

Such that $\bar{f}(\lambda^t) = \left[\bar{f}_1(\lambda^t),\ldots,\bar{f}_k(\lambda^t),\ldots,\bar{f}_s(\lambda^t)\right]^T$

and $\tilde{f}(\lambda^t) = \left[\tilde{f}_1(\lambda^t),\ldots,\tilde{f}_i(\lambda^t),\ldots,\tilde{f}_m(\lambda^t)\right]^T$

Step 2: During the second phase, we find the set of initial weighting parameters for all DMUs, and reach the first answer of the decision variables.

Suppose for given positive weight vectors $w^t = \{w_1^t,\ldots,w_k^t,\ldots,w_s^t\}$ and $v^t = \{v_1^t,\ldots,v_i^t,\ldots,v_m^t\}$.

Now minimax problem can be $\lambda^t = \{\lambda_{11}^t,\ldots,\lambda_{jr}^t,\ldots,\lambda_{nn}^t\}$,

In addition, we can discover the optimal value:

$w_k(\bar{f}_k^* - \bar{f}_k(\lambda)) = \gamma_k$, $v_i(\tilde{f}_i(\lambda) - \tilde{f}_i^*) = \tau_i$ are given by β_k^t and α_i^t

Iteration $t = 0$. Select an initial point $\lambda^0 = \left[\lambda_{11}^0,\ldots,\lambda_{nn}^0\right]$ and

$$v_i^t = \left(\frac{1}{X_i^* - f''_i}\right) \quad \forall i = 1,\ldots,m.$$ and

$$w_k^t = \left(\frac{1}{f'_k - Y'^*_k}\right) \quad \forall k = 1,\ldots,s.$$

Now $\beta^t = \left[\beta_1^t,\ldots,\beta_s^t\right]^T$, $\alpha^t = \left[\alpha_1^t,\ldots,\alpha_m^t\right]^T$ can be the first dual variable values.

Step 3: The normal vectors N^t at $\bar{f}(\lambda^t)$ and M^t at $\tilde{f}(\lambda^t)$ on the efficient frontier are indicated by $N^t = \left[w_1^t\beta_1^t,\ldots,w_k^t\beta_k^t,\ldots,w_s^t\beta_s^t\right]^T$ and $M^t = \left[v_1^t\alpha_1^t,\ldots,v_i^t\alpha_i^t,\ldots,v_m^t\alpha_m^t\right]^T$

After generating reference total output and input, we can then select the most preferred solution and the interactive process will be finished. otherwise, the decision maker (DM) is asked to make local indifference trade-offs on objectives.

Step 4: Determine the trade-off direction.

It is assumed that $\bar{f}_1$ and $\widetilde{f}_1$ are reference objectives. G^t_{1k} and R^t_{1i} are indifference trade-offs, and the marginal rates of substitution G^t at $\bar{f}(\lambda^t)$ and R^t at $\widetilde{f}(\lambda^t)$ are as follows:

$$G^t_{1k} = -\frac{d\bar{f}^t_1}{d\bar{f}^t_k} \quad and \quad G^t = \left[1, G^t_{12}, ..., G^t_{1k}, ..., G^t_{1s}\right]^T$$

$$R^t_{1i} = -\frac{d\widetilde{f}^t_1}{d\widetilde{f}^t_i} \quad and \quad R^t = \left[1, R^t_{12}, ..., R^t_{1i}, ..., R^t_{1m}\right]^T$$

It is worth noting that the projection Δu^t and Δs^t can be discovered by using G^t and R^t.

$$\Delta\bar{u}^t = \left[\Delta\bar{f}'^t_1, ..., \Delta\bar{f}'^t_k, ..., \Delta\bar{f}'^t_s\right] = -G^t + \frac{\left[(G^t)^T N^t\right]}{\left[(N^t)^T N^t\right]} N^t$$

$$\Delta s^t = \left[\Delta\bar{f}''^t_1, ..., \Delta\bar{f}''^t_i, ..., \Delta\bar{f}''^t_m\right] = R^t - \frac{\left[(R^t)^T M^t\right]}{\left[(M^t)^T M^t\right]} M^t$$

Step 5: Calculate the desired step sizes.

The step sizes are to be determined by the largest and smallest desired step. We use $\bar{\gamma} = \bar{\gamma}^t_{\max}\gamma$ and $\bar{\tau} = \bar{\tau}^t_{\min}\tau$ to show how $\bar{\gamma}^t_{\max}$, $\bar{\tau}^t_{\min}$ are suitable for the description of the largest permissible step sizes. Here follows how the maximum step size can be estimated:

$$s^t_{\min} = \bar{\tau}^t_{\min} = \min_{i \in J^1} \frac{\widetilde{f}^t_i - f''_i}{\left|\Delta f''_i\right|} \quad , \quad s^t_{\max} = \bar{\gamma}^t_{\max} = \max_{k \in I^1} \frac{f'_k - \bar{f}^t_k}{\left|\Delta f'^t_k\right|}$$

The step size s^t is taking interval $\left[0, s^t_{\max}\right]$ and given that

$$s^t_l = \frac{l}{C_s} . s^t_{\max} \qquad l = 1,2..., C_S$$

The step size s^t is taking interval $\left[s^t_{\min}, 0\right]$ and given that

$$s^t_l = \frac{l}{C_s} . s^t_{\min} \qquad l = 1,2..., C_S$$

Regarding C_S , it indicates an integer that could be composed to modify accuracy of s^t

At this point, vectors of worth can be estimated,

$$\left(f^t_1 + s^t_1 . \Delta f^t_1, ..., f^t_s + s^t_l . \Delta f^t_s\right) \qquad l = 1 ... C_s$$

Step 6: The new reference point is defined by $\bar{q}^t = (\bar{q}_1^t, ..., \bar{q}_s^t)$ and $\tilde{q}^t = (\tilde{q}_1^t, ..., \tilde{q}_m^t)$

Where $\tilde{q}_i^t = \tilde{f}_i^t + \tilde{\tau}^t.\Delta f_i''^t \qquad \forall i = 1...m$

$$\bar{q}_k^t = \bar{f}_k^t + \bar{\gamma}^t.\Delta f_k'^t \qquad \forall k = 1...s$$

And $\tilde{\mu}_i^t = \dfrac{1}{|\tilde{q}_i^t - \tilde{f}_i^t|} \qquad \forall i = 1...m \;\; \bar{\mu}_k^t = \dfrac{1}{|\bar{q}_k^t - \bar{f}_k^t|} \qquad \forall k = 1...s$

Let (λ^*, f^*) be a solution.

Step 7: select the most preferred solutions by the decision maker.

First we determined the solution (λ^*, f^*), if $u(f^*) \geq u(f^t)$, then $\lambda^{t+1} = \lambda^*$, and $f^{t+1} = f^*$.

When these steps have been completed, we are now ready to $t = t + 1$ and go to Step 2. otherwise:

$$\gamma^t := \gamma_{old}{}^t.\min\left\{\left|1 - \frac{\|\bar{q}_k{}^t - f'^*\|_2}{\|\bar{q}_k{}^t - \bar{f}^t\|_2}\right|, \frac{1}{2}\right\} \leq \frac{1}{2} s_{old}^t$$

$$\tau^t := \tau_{old}{}^t.\min\left\{\left|1 - \frac{\|\tilde{q}_i{}^t - f''^*\|_2}{\|\tilde{q}_i{}^t - \tilde{f}^t\|_2}\right|, \frac{1}{2}\right\} \leq \frac{1}{2} s_{old}^t$$

Decision maker will select the best solutions, otherwise, we will go to Step 6.

Numerical Illustration

The following table summarizes the data pertaining to seven banks. These data were prepared as described by Yang (2004), the sample was selected on the basis of four inputs and two outputs. This sample was solved by Project algorithm, the method has several benefits, for instance, it searches for the MPS on the efficient frontier. We noted from Table 1 that four inputs are branches, number of ATMs, staff and asset size. The two outputs are customer satisfaction and total revenue. On the other hand, for this study, it is assumed that the data have been changed from certain to imprecise, as shown in Table 2.

Table 1. Original Data Set

DMU	Bank	Branches	ATMs	Staff	Asset size	Customer	Total
1	Abby	0.77	2.18	2.35	2.96	6.79	10.57
2	Barclay	1.95	3.19	8.43	3.53	2.55	13.35
3	Halifax	0.80	2.10	3.21	2.41	9.17	8.14
4	HSBC	1.75	4.00	13.30	4.85	5.82	23.67
5	Lloyds TSB	2.50	4.30	9.27	2.40	6.57	14.01
6	Nat West	1.73	3.30	7.70	3.09	4.86	12.04
7	RBS	0.65	1.73	2.65	1.34	7.28	7.36

Table 2. Interval Data Set

DMU	Bank	Branch	ATMs	Staff	Asset size	Customer	Total
1	Abby	0.77	[2.18,2.2]	[2.35,2.50]	[2.96,3]	6.79	10.57
2	Barclay	1.95	[3.19,3.20]	[8.43,8.50]	[3.53,4]	2.55	13.35
3	Halifax	0.80	[2.10, 2.2]	[3.21,3.50]	[2.41,2.5]	9.17	8.14
4	HSBC	1.75	[4.00,4.1]	[13.3,13.5]	[4.85,5]	5.82	23.67
5	Lloyds TSB	2.50	[4.30,4.7]	[9.27,9.3]	[2.40,2.43]	6.57	14.01
6	Nat West	1.73	[3.30,3.40]	[7.7 ,7.8]	[3.09,3.2]	4.86	12.04
7	RBS	0.65	[1.73,1.80]	[2.65,2.7]	[1.34,1.4]	7.28	7.36

The software application by which we run target Model (1) , and used to analyze the data was Lingo. By the Project method, we are able to reach the MPS. we believe this solution will aid researchers to find more decrease of all inputs and more increase of all outputs, thereby yielding in saving time in the calculation of the method. The main difference of the suggested method in this paper was the imprecise data.

Conclusions

In conclusion, we obtained an equivalence relation between the imprecise data envelopment scenario analysis and the minimax MOLP formulation. These observations might help to solve the imprecise DESA model. The results of this study suggest that interactive Project algorithm is suitable for solving the imprecise DESA problem and our method could achieve the MPS. In addition, we believe that our results may improve knowledge about obtaining any efficient solution. Also, this approach results in decreasing total input and increasing total output at the same time. Also, it handles one problem instead of solving n independent linear programming models.

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