

Analysis of a Dominant Retailer Dual-Channel Supply Chain under Customers' Channel Preference: The α -Branch and Bound Solution Algorithm

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Abstract

This paper investigates the Stackelberg equilibrium for pricing and ordering decisions in a dominant retailer dual-channel supply chain. In a dual-channel sales network, the products are sold through both a traditional, physical retailer and a direct Internet channel. It is assumed that the retailer is the leader and the powerful member of the supply chain has the market power and acts as a leader and proposes his/her inventory policies and dollar-markup and the manufacturer, as a follower, will decide on the wholesale price and the price of Internet channel as well as the inventory quantity of online store based on the retailer's decisions. The situation is formulated as a bi-level programming problem, and it is converted to a single level model using Karush-Kuhn-Tucker (KKT) conditions. The single level problem is solved using the α -Branch and Bound (α -BB) algorithm. We investigate the significance of customers' channel preference on adopting an online channel by the manufacturer. We show that an online channel is not always detrimental to a retailer, but in a Pareto-zone, a range or zone of customers' channel preference, both supply chain members benefit from newly added sales channel.

Keywords

Retail supply chain; Dual-channel supply chain; Channel completion; Dominant Retailer; α -Branch and Bound.

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Introduction

Today, the retailers are often much more powerful than manufacturers, and as a result they have more influence on whole supply chain procedures. Wal-Mart is a good example of a dominant retailer that constantly employs pressure on its suppliers to reduce their prices and improve service level. In today's market, in addition to the dominant retailers, there are two different categories of customers: I) some customers prefer online shopping. These kinds of customer dislike shopping from traditional, physical stores and II) other customers dislike online shopping and prefer to take their purchases home immediately. When a product has a high customers' channel preference, it will have a higher possibility of being sold on the Internet. To attract both types of consumers, many distributors sell their products through a dual-channel sales network, joining the physical channel with online sales.

This paper examines a dual-channel supply chain, a physical retailer with a manufacturer-owned online store. We assume that the retailer has the market power and could influence the pricing policy of the manufacturer. The problem has a Stackelberg game structure in which the retailer and the manufacturer are the leader and the follower, respectively. The hierarchical structure of the model results in a bi-level programming problem that is solved using Karush-Kuhn-Tucker conditions and α -Branch and Bound (α -BB) algorithm. We find the Stackelberg equilibrium for the wholesale and retail prices as well as inventory quantities in a single-period setting. Our model incorporates customers' channel preference for purchasing channel through random demand functions. The results show that in a Pareto-zone, a range or zone of customers' channel preference, both supply chain members benefit from newly added online sales channel.

At last, we carried out a sensitivity analysis of some of the model parameters and investigated their effects on the equilibrium.

Literature Review

Recently, a vast majority of the literature has considered the interaction and competition between the retailer and the manufacturer and their effect on operational decisions in a dual-channel supply chain. The manufacturers in a dual-channel supply have two revenue

sources: the revenue of selling to the physical retailer and the revenue of selling directly to customers. However, the result is not always beneficial to the manufacturers because of channel conflict (Xiao, Choi, & Cheng, 2014). The manufacturer acts as both a supplier and a retailer. This is, in fact, a reason of conflict between supply chain members (Tsay & Agrawal, 2004).

In supply chains with a powerful dominant retailer, the manufacturer, in order to increase his/her sales and control the dominant retailer, adds a direct online channel and shapes a dual-channel supply chain (J. Chen, Zhang, & Sun, 2012). As mentioned in Modak and Kelle (2018), the main aim behind using a dual-channel supply chain is to cover a wider range of customers.

Chiang et al. (2003) use a Stackelberg game and provide a strategic analysis to investigate the interactions between the manufacturer and the retailer. Chen et al. (2008) investigate the influence of service level on each of sales channels in a dual-channel and finally decide when the manufacturer should create a direct channel. Hua et al. (2010) adopt that three factors affect the demand function of each sales channel: the price of physical retailer, the price of online channel and at last the quoted lead time. They analyze the impacts of the delivery lead time of the direct channel on the price decision of each channel and the profits of the manufacturer and the retailer. Panda et al. (2015) consider a sale network for high-tech products and obtain pricing and replenishment policies and conclude that product compatibility with online sales has a significant impact on the pricing policy. Xu et al. (2012) extend the work of Chiang et al., (2003) and consider the delivery lead time decision in the dual-channel supply chain model. Chen et al. (2017) integrate the channel environmental sustainability into the dual-channel supply chain and discuss the environmental sustainability strategies and pricing policies simultaneously. Mukhopadhyay et al. (2008) consider a mixed-channel model that the retailer is allowed to add value to the product to differentiate its offering for the customers. They obtain optimum pricing decisions, the amount of value added to the product by the retailer, and the manufacturer's wholesale price for the retailer. They incorporate information asymmetry, where the manufacturer has incomplete information about the retailer's cost of adding value.

Batarfi et al. (2016) consider a structure that standard and customized products are sold through the physical retailer and online sales network, respectively. They include sales prices, the delivery lead-time and product differentiation as three factors that impact the demand function and identify the optimal policy to maximize the total profit of the supply chain. Zhang et al. (2012) consider three different power structures (which are manufacturer Stackelberg, Retailer Stackelberg and Vertical Nash) and investigate the impact of product substitutability and pricing strategies in a dual exclusive channel system. Li et al. (2014) consider a dual channel with a risk-neutral manufacturer and a risk-averse retailer and investigate the equilibrium results and show that the retail price will decrease as the retailer becomes more risk averse.

The above cited literature mostly focuses on studying the optimal operational decisions in a dual-channel supply chain. However, collaboration and coordination mechanisms of dual sales have attracted much attention in the past few years. These studies usually use a supply chain contract to decline the channel conflict. These contracts usually enable the physical retailer to set pricing and ordering policies that are equivalent to those in an integrated supply chain. Coordination between the retailer and the manufacturer of the supply chain can make a Pareto improvement in profits.

David and Adida (2015) propose a linear quantity discount contract to coordinate the dual channel supply chain. They consider a single supplier that both controls an online channel and sells its product through multiple differentiated retailers. Chen et al. (2012) show that a contract that determines the pricing policy of a manufacturer with a complementary agreement, such as profit-sharing or two-part tariff agreement, could coordinate the supply chain and reduce the conflict between sales channels. Dumrongsiri et al. (2008) investigate when the manufacturer benefits more from adding a new Internet channel. They conclude that this situation occurs when the retailer's marginal cost is high and the variability of wholesale price and demand is low. Cattani et al. (2006) analyze price homogeneity between the two channels to abate the channel conflict. They found that this strategy increases profits for both the retailer and the manufacturer, the supplier benefiting from additional revenue and the retailer benefiting from a wholesale

price reduction. Yan et al. (2011) present that brand differentiation cannot coordinate the sales network. They observed that when the manufacturer sets the wholesale price and online price in a way that maximizes the whole supply chain, a profit sharing contract could coordinate the sales network. Chen et al. (2013) investigate brand loyalty in a dual-channel supply chain. They found out that improving brand loyalty was profitable for both of the manufacturer and retailer, and that an increased service value may alleviate the threat of the Internet channel for the retailer and increase the manufacturer's profit. Zhao et al. (2009) investigate coordination mechanisms under symmetric information and asymmetric information in a single dominant retail-channel. They implement an option contract to coordinate the sale network.

Tsay and Agrawal (2004) show that adding an online channel is not always detrimental to the traditional retailer, but also it may be beneficial for both the retailer and the manufacturer because of counteracting double marginalization. They suggest that coordination of the supply chain may be achieved by giving the reseller a commission for diverting customers toward the online sales channel. The result of Cai (2010) is the same. He introduces the channel-adding Pareto-zone, a range or zone of customers' channel preference, in which both supply chain members benefit from newly added online sales channel. Xu et al. (2018) consider the coordination of a dual-channel supply chain under mandatory carbon emission capacity regulation.

In general, the dual-channel problem is modeled by game theory and bi-level programming and the solution methods used to tackle these problems are not vast. Most of the papers in this area are deterministic bi-level problems. To solve such a problem, the optimization problem of the lower-level problem is solved by considering leaders' decisions as parameters and in the next step, the upper-level problem obtains the optimal value of decision variables using the optimal values that are computed in the lower-level problem. We can mention Cai (2010); Cattani et al. (2006); Dumrongsiri et al. (2008); Hua et al. (2010); Li et al. (2014); Modak & Kelle,(2018); Tsay & Agrawal (2004) as some examples that tackle their problems by this solution method. Chen et al. (2012) use Lagrangian

Relaxation method to obtain the optimal solutions of prices in both retail and online channels.

The impact of the customers' channel preference on the performance of supply chain members in a *retailer-Stackelberg dual-channel supply chain* setting with random demand functions is not studied in the literature and we found it as a literature gap. In this paper, we consider customers' channel preference and price-sensitivity at the same time to analyze the supply chain performance. We show that, even in a “retailer-Stackelberg scenario”, there is a range of the customers' channel preference in which supply chain members benefit from the new online store.

As stated earlier, Cai (2010) studies a manufacturer-Stackelberg situation and shows the existence of a Pareto-zone. Our paper explores the same feature in a retailer-Stackelberg scenario. In particular, we show that even when the retailer is the dominant member of the supply chain, there are situations where both the manufacturer and the retailer benefit from dual-channel supply chain. It is worth noting that the solution method of our paper is completely different of Cai (2010).

Mathematical modeling

In this section, we introduce the notation and formulation used in our dual-channel supply chain problem. As shown in Figure 1, we consider a market with a traditional retailer that buys a product from a manufacturer. The manufacturer sells his/her product through both retail channel and a newly added online store. We consider a linear demand model with an uncertain additive part. Our problem involves a hierarchical decision-making process and fits well to a Stackelberg game that is formulated as a bi-level programming problem.

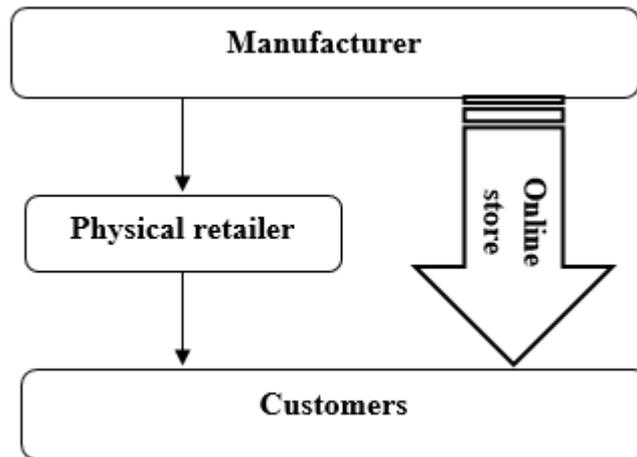


Fig. 1. A dual-channel supply chain

In the whole paper, we use subscripts and superscripts as follow:

Table 1. Subscripts and Superscripts

Subscripts	
m	the manufacturer
d	Online store
r	Physical retailer
Superscripts	
SRS	Single retail channel
DRS	Dual retailer Stackelberg

Modeling of dominant retailer single channel supply chain

In the single channel setting, the manufacturer distributes the products through just a physical retailer. In this scenario, the retailer announces his/her dollar-markup m_r . As a consequence, for whatever wholesale price that is quoted by the manufacturer, the unit retail price in the market will be $p_r = w + m_r$.

Market's demand function is defined as $D_r(p_r) = \delta - \alpha p_r + \varepsilon_r$ where δ is the base and the potential market size of the product and α is the sensitivity of the demand to the price.

In the single channel supply chain, the retailer takes the manufacturer's response function into account for declaring his/her own dollar-markup and inventory policies. The problem has a

hierarchical structure and is modeled as a bi-level programming problem as follow:

Upper-level problem/ retailer problem:

$$\begin{aligned} \text{Max}_{m_r, z_r} E [\Pi_r(p_r, z_r)] = & m_r (\delta - \alpha(w + m_r) + \mu_r) - (m_r + s_r) \Theta_r(z_r) \\ & - (w - v_r) \Lambda_r(z_r) \end{aligned} \quad (1)$$

S.T:

$$m_r = p_r + w, p_r \geq w, a_r \leq z_r, z_r \leq b_r, p_r \geq p_r^{\min}, p_r \leq p_r^{\max}$$

Lower-level problem/ manufacturer problem:

$$\begin{aligned} \text{MAX}_w E [\Pi_M(w, p_d, z_d)] = & \text{MAX}_w E [\Pi_m(w)] = (w - c) (\delta - \alpha(w + m_r) + z_r) \end{aligned} \quad (2)$$

S.T:

$$c \leq w$$

In the upper-level problem $m_r = p_r - w \geq 0$ guarantees the profitability of the retailer and in the lower level $c \leq w$ does the same for the manufacturer. Other constraints are necessary bounds.

Demand function modeling

In a dual channel supply chain, the product is sold through both a traditional physical retailer and an online store. We segment the market with size (δ) into two submarkets through the consideration of customers' channel preference: $\delta_r = a\delta$ represents the size of the submarket that prefers to buy from physical retailer and $\delta_d = (1-a)\delta$ represents the size of the submarket that prefers the online store where $a \in [0,1]$. The value of the parameter a is called "customers' channel preference". More information about customers' channel preference in electronic markets can be found in studies by Kacen, Hess, & Chiang, (2013) and Liang & Huang (1998).

We use a downward sloping linear demand function with an additive uncertain part which is used broadly in the literature (Bernstein & Federgruen, 2004; F. Y. Chen, Yan, & Yao, 2004; J. Chen et al., 2012; Yan et al., 2011) and write the demand function of the traditional retailer (r) and online store (d) as follow:

$$\begin{aligned}
 D_r(p_r, p_d) &= \delta_r - \alpha_r p_r + \beta(p_d - p_r) + \varepsilon_r \\
 D_d(p_r, p_d) &= \delta_d - \alpha_d p_d + \beta(p_r - p_d) + \varepsilon_d
 \end{aligned}
 \tag{3}$$

where p_r and p_d show prices of physical retailer and online store, respectively. Assume that the set of feasible prices for channel i is a closed interval $[p_i^{\min}, p_i^{\max}] \forall i = r, d$. As mentioned earlier, the parameter δ is the base market size that is segmented into two different market shares: the base market share of retail-channel is $a\delta$ while $(1-a)\delta$ determines the base market share of online channel. The parameters $\alpha_r > 0$ and $\alpha_d > 0$ ($\alpha_r > \beta$, $\alpha_d > \beta$) are price sensitivities in the retail and online stores, respectively. The parameter $\beta \geq 0$ is cross-price sensitivity and shows substitutability of the products that are distributed in two channels. This parameter also shows the size of the market that shifts from the retail channel to the online store per unit increase in the price difference between p_r and p_d when $p_r > p_d$. The random variable $\varepsilon_i \forall i = r, d$ indicates the uncertain part of the demand function of channel i . Probability density function (PDF), cumulative distribution function (CDF), and expected value of ε_i are assumed known and are denoted by $f_i(\cdot)$, $F_i(\cdot)$ and π_i . Assume that the range of ε_i is $[a_i, b_i] \forall i = r, d$.

Define the deterministic part of each demand function by $\gamma_i(p_i, p_{-i}) = \delta_i - \alpha_i p_i + \beta(p_{-i}, p_i) \forall i = r, d$. p_{-i} shows the price of channel i 's competitor. Notice that: (I) $\frac{\partial \gamma_i}{\partial p_i} = -\alpha_i - \beta < 0$ which means the demand function of channel i is downward sloping in his/her price and (II) $\frac{\partial \gamma_i}{\partial p_{-i}} = \alpha_{-i} + \beta > 0$ which means when channel i 's competitor increases his/her price, the demand of channel i will strictly increase and at last (III) $\gamma_i(p_i, p_{-i})$ takes its maximum (minimum) value when the channel i chooses p_i^{\min} (p_i^{\max}) and its competitor chooses p_{-i}^{\max} (p_{-i}^{\min}).

Total deterministic market demand is $D_T(p_r, p_d) = \delta - \alpha_r p_r - \alpha_d p_d$. With one unit increase of price in two channels, we will have α unit decrease demand (where $\alpha = \alpha_r + \alpha_d$). If we define $k = \frac{\alpha_r}{\alpha}$ and $1 - k = \frac{\alpha_d}{\alpha}$ as relative price sensitivity in the retail and direct online channels as respectively,

the demand function of each physical and online channel would be as follow:

$$\begin{cases} D_r(p_r, p_d) = a\delta - \underbrace{\alpha k p_r + \beta(p_d - p_r)}_{\gamma_r} + \varepsilon_r \\ D_d(p_r, p_d) = (1-a)\delta - \underbrace{\alpha(1-k)p_d + \beta(p_r - p_d)}_{\gamma_d} + \varepsilon_d \end{cases} \quad (4)$$

Modeling of a dominant-retailer dual-channel supply chain

In this section, we study a supply consisting of a manufacturer and a dominant retailer. The manufacturer decides to open a direct online channel. We consider that the two channels compete in a market with a potential size (a). With a new online store, a percentage of customers now prefer to buy the product from the newly added online channel. In contrary to normal situations that the manufacturer is market leader and chooses the wholesale price, in the retailer-Stackelberg setting the manufacturer chooses the wholesale price after that the retailer announces his/her profit margin.

In this section, we consider a supply chain consisting of a manufacturer who distributes his/her product through two channels: an independently physical retailer and a manufacturer-owned online store. It is assumed that the retailer is the powerful member of the supply chain. In this case, the sequence of decisions is as follows:

1. The retailer declares his profit margin m_r and orders q_r from the manufacturer.
2. The manufacturer sets the wholesale price w charging the retailer. He/ She knows that, for whatever w he/she quotes, the unit price of physical retailer will be $p_r = m_r + w$. At the same time, the manufacturer also determines the online store price p_d and chooses a production quantity to be shipped to the online store q_d .
3. The retailer uses response function of the manufacturer and chooses the best markup to maximize his/her profit.
4. The manufacturer will then produce the total number of ordered quantities $Q = q_d + q_r$ with a unit production cost of c .
5. There is a shortage cost $s_i \forall i=r,d$ for each unit of unsatisfied demand in different sale stores.

6. Excess inventory of each channel is salvaged with the value of $v_i \forall i=r,d$.

To simplify the model analysis, like Petruzzi and Dada (1999), we define $z_i \forall i=r,d$ as the amount of safety stock that is held to confront the demand randomness and write $q_i = \gamma_i + z_i$ where γ_i is the deterministic part of the demand function that was defined earlier. Notice that the range of z_i is $[a_i, b_i]$.

With this reformulation, the decision variables for the retailer are m_r and z_r while the manufacturer decides about w, p_d and z_d .

The expected profit function of the retailer is similar to a newsvendor model with price and order quantity as decision variables that are written as follows:

$$E[\Pi_r(p_r, q_r, D_r)] = m_r(\mu_r + \gamma_r) - (m_r + s_r)\Theta_r(z_r) - (w - v_r)\Lambda_r(z_r) \tag{5}$$

In the above-mentioned profit function, $\Lambda_r(z_r)$ and $\Theta_r(z_r)$ are expected excess and shortage inventory, which are obtained as follow:

$$\Lambda_r(z_r) = \int_{a_r}^{z_r} F_r(\varepsilon_r) d\varepsilon_r \text{ and } \Theta_r(z_r) = \mu_r - z_r + \Lambda_r(z_r). \text{ For more details, see}$$

Appendix 1.

To have the retailer price in its explicit form, we can substitute m_r with $p_r + w$ in the profit function.

The manufacturer has two different revenue sources: (1) the profit that he/she earns by selling the product to the physical retailer and (2) the profit that he/she acquires by selling the product directly to customers through the online store.

The expected profit function of the manufacturer is written as follows:

$$E[\Pi_M(w, p_d, z_d)] = (w - c)(z_r + \gamma_r) + (p_d - c)(\mu_d + \gamma_d) - (p_d + s_d - w)\Theta_d(z_d) - (w - v_d)\Lambda_d(z_d) \tag{6}$$

Expected excess and shortage inventory in the online store $\Lambda_d(z_d)$ and $\Theta_d(z_d)$ are defined like the ones for the physical retailer:

$$\Lambda_d(z_d) = \int_{a_d}^{z_d} F_d(\varepsilon_d) d\varepsilon_d \text{ and } \Theta_d(z_d) = \mu_d - z_d + \Lambda_d(z_d).$$

In the dual retailer-Stackelberg supply chain, to obtain his/her best markup decision, the retailer uses the manufacturer's reaction function into account. To model the retailer-Stackelberg game, which has a hierarchical structure, we develop a bi-level model as follow:

Upper-level problem/ retailer problem:

$$\begin{aligned} \text{Max}_{p_r, z_r} E[\Pi_r(p_r, q_r)] = & (p_r - w)(\mu_r + \gamma_r) - (p_r + w + s_r)\Theta_r(z_r) \\ & - (w - v_r)\Lambda_r(z_r) \end{aligned} \quad (7)$$

S.T:

$$p_r \geq w, a_r \leq z_r, z_r \leq b_r, p_r \geq p_r^{\min}, p_r \leq p_r^{\max}$$

Lower-level problem/ manufacturer problem:

$$\begin{aligned} \text{MAX}_{p_d, z_d, w} E[\Pi_M(w, p_d, z_d)] = & (w - c)(z_r + \gamma_r) + (p_d - c)(\mu_d + \gamma_d) \\ & - (p_d + s_d - w)\Theta_d(z_d) - (w - v_d)\Lambda_d(z_d) \end{aligned} \quad (8)$$

S.T:

$$c \leq w, w \leq p_d, a_d \leq z_d, z_d \leq b_d, p_d \geq p_d^{\min}, p_d \leq p_d^{\max}$$

In the upper level problem, to ensure the profitability of the retailer, we require $m_r = p_r - w \geq 0$; also the prices must be in the range $[p_r^{\min}, p_r^{\max}]$. In the lower level problem, the constraint $c \leq w$ is to guarantee the profitability of the manufacturer; also we require $w \leq p_d$ preventing the retailer from buying his/her needed products from the online store rather than through the manufacturer. The amount of safety stock in channel $i \forall i=r, d$ needs to be in the range $[a_i, b_i]$, based on the assumptions in both problems.

Solution approach to the dual retailer-Stackelberg problem

The optimization problem of the lower-level problem considers m_r and z_r as parameters and obtains the optimal values of selling price p_d , safety stock z_d and wholesale price w which depend on m_r and z_r . The upper-level problem obtains the optimal value of m_r and z_r using the optimal values p_d, z_d and w that are computed in the lower-level problem. It is not possible to solve the bi-level problem (5)-(6) in this form. A common approach to solve a bi-level problem, as stated by Colson et al. (2005), is to replace Karush-Kuhn-Tucker (KKT)

conditions of the lower-level problem when it is a convex optimization and yields a single-level reformulation of the problem.

The objective function of our lower-level problem is a maximization one; then to use KKT conditions, it is enough to show that it is joint concave in its decision variables p_d , z_d and w .

Lemma 1. The manufacturer profit function in the lower level problem of dual retailer-Stackelberg model is joint concave of variables p_d , z_d and w .

Proof.

A multivariate function $f(x_1, x_2, \dots, x_n)$ with continuous partial derivatives and cross partial derivatives on a convex open set S is concave if and only if its hessian matrix $H(x)$ is negative semidefinite for all $x \in S$. Following this rule, it is enough to show that: $(-1)^k D_k > 0 \forall k = 1, 2, 3$ where D_k is k -order leading principal minor of hessian matrix. We form the hessian matrix of manufacturer profit function as follow:

$$H(p_d, w, z_d) = \begin{bmatrix} -2\alpha(1-k) - 2\beta & 2\beta & 1 - F_d(z_d) \\ 2\beta & -2\alpha k - 2\beta & -1 \\ 1 - F_d(z_d) & -1 & (p_d + s_d - v_d)f_d(z_d) \end{bmatrix} \quad (9)$$

We write the hessian matrix of manufacturer profit as sum of two hessian matrices as follow:

$$H(p_d, w, z_d) = H_1(p_d, w, z_d) + H_2(p_d, w, z_d)$$

where

$$H_1(p_d, w, z_d) = \begin{bmatrix} -2\alpha(1-k) - 2\beta & 2\beta & 1 - F_d(z_d) \\ 2\beta & -2\alpha k - 2\beta & -1 \\ 1 - F_d(z_d) & -1 & (p_d + s_d - v_d)f_d(z_d) \end{bmatrix},$$

$$H_2(p_d, w, z_d) = \begin{bmatrix} -2\beta & 0 & 0 \\ 0 & -2\beta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The sum of two negative semidefinite matrices is still negative semidefinite; then it is enough to show that both $H_1(p_d, w, z_d)$ and $H_2(p_d, w, z_d)$ are negative semi-definite.

For $H_1(p_d, w, z_d)$, we have:

$$(-1)^1 D_1 = 2\alpha_d > 0$$

$$(-1)^2 D_2 = \begin{vmatrix} -2\alpha(1-k) & 2\beta \\ 2\beta & -2\alpha k \end{vmatrix} = 4(\alpha^2 k(1-k) - \beta^2) \xrightarrow{ak > \beta, \alpha(1-k) > \beta} (-1)^2 D_2 > 0$$

$$\begin{aligned} (-1)^3 D_3 &= \begin{vmatrix} -2\alpha_d & 2\beta & F_d(z_d) - 1 \\ 2\beta & -2\alpha_r & 1 \\ F_d(z_d) - 1 & 1 & (p_d + s_d + v_d)f_d(z_d) \end{vmatrix} \\ &= (-1)^3 \left[\begin{aligned} &(4\alpha^2 f_d(z_d)k^2 - 4\alpha^2 f_d(z_d)k + 4\beta^2 f_d(z_d))(p_d + s_d + v_d) \\ &+ 2(F(z_d))^2 \alpha k - 4F_d(z_d)\alpha k + 4F_d(z_d)\beta + 2\alpha - 4\beta \end{aligned} \right] \\ &= (-1)^3 \left[\begin{aligned} &\underbrace{(p_d + s_d + v_d)4f_d(z_d)(\alpha^2 k^2 + \beta^2 - \alpha^2 k)}_{\leq 0 \text{ because: } \alpha^2 k^2 + \beta^2 - \alpha^2 k = -\alpha^2 k(1-k) + \beta^2 < -\alpha^2 + \beta^2 < 0} \\ &+ 2\alpha_r F_d^2(z_d) - 4\alpha_r F_d(z_d) + 4\beta_d F_d(z_d) + 2\alpha_d + 2\alpha_r - 4\beta \end{aligned} \right] \leq 0 \\ &\leq 0 \text{ if } \frac{\beta}{\alpha} \geq \frac{k(F(z_d))^2 - 2kF(z_d) + 1}{-2(F-1)} \xrightarrow{\text{always } \frac{k(F(z_d))^2 - 2kF(z_d) + 1}{-2(F-1)} \geq 0} \text{The whole above term} \leq 0 \end{aligned}$$

Then, $H_1(p_d, w, z_d)$ is negative semi-definite. For $H_2(p_d, w, z_d)$, we have:

$$(-1)^1 D_1 = 2\beta > 0$$

$$(-1)^2 D_2 = \begin{vmatrix} -2\beta & 0 \\ 0 & -2\beta \end{vmatrix} = 4\beta^2 > 0$$

$$(-1)^3 D_3 = \begin{vmatrix} -2\beta & 0 & 0 \\ 0 & -2\beta & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Then, $H_2(p_d, w, z_d)$ is negative semi-definite.

Proof is complete.

As is shown in Lemma 1, the lower level problem is a concave maximization problem in terms of the variables p_d , w and z_d . As a result, we write the Karush–Kuhn–Tucker optimality conditions to

convert the original bi-level model (5)-(6) into an equivalent single-level optimization model.

A necessary and sufficient condition for p_d , w and z_d to be optimal solutions to the competitor's problem is that there exist Lagrangian multipliers $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ which satisfy the following system:

$$\begin{aligned}
 \beta(w - c) + \mu_d + \gamma_d + (p_d - c)(-\alpha(1 - k)) - \Theta_d(z_d) &= -\lambda_2 - \lambda_5 + \lambda_6 \\
 z_r + \gamma_r + (w - c)(-\alpha k - \beta) + \beta(p_d - c) + \Theta_d(z_d) - \Lambda_d(z_d) &= -\lambda_1 + \lambda_2 \\
 -(p_d + s_d - w)(F(z_d) - 1) - (w - v_d)F(z_d) &= -\lambda_3 + \lambda_4 \\
 \gamma_r &= a\delta - \alpha k p_r + \beta(p_d - p_r) \\
 \gamma_d &= (1 - a)\delta - \alpha(1 - k)p_d + \beta(p_r - p_d) \\
 \lambda_1(c - w) &= 0 \\
 \lambda_2(w - p_d) &= 0 \\
 \lambda_3(a_d - z_d) &= 0 \\
 \lambda_4(z_d - b_d) &= 0 \\
 c \leq w, w \leq p_d, a_d \leq z_d, z_d \leq b_d &
 \end{aligned} \tag{10}$$

Using the system of (10), we write the single-level problem of dual retailer-Stackelberg supply chain as follow:

Single-level Problem:

$$\begin{aligned}
\text{Max } E[\Pi_r(p_r, q_r, D_r)] &= (p_r - w)(\mu_r + \gamma_r) - (p_r + s_r - w)\Theta_r(z_r) - (w - v_r)\Lambda_r(z_r) \\
\text{S.T} \\
\beta(w - c) + \mu_d + \gamma_d + (p_d - c)(-\alpha(1 - k)) - \Theta_d(z_d) &= -\lambda_2 - \lambda_5 + \lambda_6 \\
z_r + \gamma_r + (w - c)(-\alpha k - \beta) + \beta(p_d - c) + \Theta_d(z_d) - \Lambda_d(z_d) &= -\lambda_1 + \lambda_2 \\
-(p_d + s_d - w)(F(z_d) - 1) - (w - v_d)F(z_d) &= -\lambda_3 + \lambda_4 \\
\gamma_r &= a\delta - \alpha k p_r + \beta(p_d - p_r) \\
\gamma_d &= (1 - a)\delta - \alpha(1 - k)p_d + \beta(p_r - p_d) \\
\lambda_1(c - w) &= 0 \\
\lambda_2(w - p_d) &= 0 \\
\lambda_3(a_d - z_d) &= 0 \\
\lambda_4(z_d - b_d) &= 0 \\
\lambda_5(p_d^{\min} - p_d) &= 0 \\
\lambda_6(p_d - p_d^{\max}) &= 0 \\
\lambda_i &\geq 0 \quad \forall i = 1, 2, \dots, 6 \\
m_r \geq 0, a_r \leq z_r, z_r \leq b_r, p_r \geq p_r^{\min}, p_r \leq p_r^{\max} \\
c \leq w, w \leq p_d, a_d \leq z_d, z_d \leq b_d, p_d \geq p_d^{\min}, p_d \leq p_d^{\max}
\end{aligned} \tag{11}$$

 α -Branch and Bound (α -BB) algorithm to solve the single-level problem

The single level formulation (11) is nonconvex due to its stationarity and complementarity constraints. For its solution, we employ the deterministic global optimization algorithm, α -Branch and Bound (α -BB). This algorithm was introduced by Androulakis et al. (1995) and then improved by Adjiman et al. (1998). The efficiency of this algorithm in relation to other methods is reported in Androulakis et al. (1995).

This algorithm guarantees convergence to a point near enough to the global minimum for the twice-differentiable nonlinear problems. The algorithm is iterative in a way that each iteration includes calculation of a lower bound and an upper bound. The lower bound is obtained by creating valid convex under-estimators for the nonconvex functions in the problem. The resulting convex nonlinear problem can be solved by global optimality. Solving the original problem locally gives an upper bound.

The convergence of α -BB algorithm to the global minimum of the problem is the result of the generation of a sequence of non-decreasing best lower bounds and non-increasing best upper bounds.

The original problem needs a preprocessing step to be prepared for the α -Branch and Bound algorithm that is discussed in detail in the next subsection.

α -Branch and Bound (α -BB) preprocessing step

In the preprocessing step of the algorithm, we need to construct *valid convex under-estimators* for all the problem functions. All problem terms are categorized into two classes: (1) terms that have a special structure such as bilinear, trilinear, fractional, fractional trilinear, univariate concave terms and (2) generic nonconvex terms, $f(x)$. Then, a convex under-estimator is generated for each term in all classes except for linear and convex terms. The relaxed problem is obtained by replacing all of its grouped functions with their valid convex under-estimators. The terms of special structure have distinctive tight convex under-estimators. However, the convexification procedure of the terms in the general nonconvex class is different from that in the other classes in the sense that it requires a more challenging and computationally severe method referred to as α calculations.

We need to notice that the solution of (11) depends on the distribution function of random variables $\varepsilon_i \forall i=r, d$. For simplicity, we assume that these random variables of demand functions are uniformly distributed within $[a_i, b_i] \forall i=r, d$. With this assumption, it is clear we would have:

$$\Theta_i(z_i) = \frac{1}{b_i - a_i} \left(\frac{b^2}{2} - bz + \frac{z^2}{2} \right) \text{ and } \Lambda_i(z_i) = \frac{1}{b_i - a_i} \left(\frac{a^2}{2} - az + \frac{z^2}{2} \right) \quad \forall i = r, d .$$

Now we convert our single level problem to a *minimization problem* and then, as stated in preprocessing step of α -BB algorithm, group all involving functions of the problem:

Single-level Problem with grouped functions (P):

$$\begin{aligned} \text{Min } E[\Pi_r(p_r, q_r, D_r)] = & \underbrace{-p_r \mu_r}_{\text{Linear}} - \underbrace{p_r \gamma_r}_{\text{Bilinear}} + \underbrace{w \mu_r}_{\text{Linear}} + \underbrace{w \gamma_r}_{\text{Bilinear}} + \underbrace{p_r \Theta_r(z_r)}_{\text{Bilinear}} \\ & - \underbrace{w \Theta_r(z_r)}_{\text{Bilinear}} + \underbrace{s_r \Theta_r(z_r)}_{\text{Linear}} + \underbrace{w \Lambda_r(z_r)}_{\text{Bilinear}} - \underbrace{v_r \Lambda_r(z_r)}_{\text{Linear}} \end{aligned} \tag{12}$$

S.T

$$\underbrace{\beta(w - c) + \mu_d + \gamma_d + (p_d - c)(-\alpha(1 - k)) - \Theta_d(z_d) + \lambda_2}_{\text{All involving terms are linear}} = 0$$

$$\underbrace{z_r + \gamma_r + (w - c)(-\alpha k - \beta) + \beta(p_d - c) + \Theta_d(z_d) - \Lambda_d(z_d) + \lambda_1 - \lambda_2}_{\text{All involving terms are linear}} = 0$$

$$-\frac{1}{b_d - a_d} \underbrace{p_d z_d}_{\text{Bilinear}} - \frac{1}{b_d - a_d} \underbrace{s_d z_d}_{\text{Linear}} + \frac{1}{b_d - a_d} \underbrace{v_d z_d + p_d + s_d - w}_{\text{Linear}} + \lambda_3 - \lambda_4 = 0$$

$$\underbrace{\gamma_r = a\delta - \alpha k p_r + \beta(p_d - p_r)}_{\text{All involving terms are linear}}$$

$$\underbrace{\gamma_d = (1 - a)\delta - \alpha(1 - k)p_d + \beta(p_r - p_d)}_{\text{All involving terms are linear}}$$

$$\Theta_d(z_d) = \frac{b_d^2}{2(b_d - a_d)} - \underbrace{\frac{b_d}{b_d - a_d} z_d}_{\text{Linear}} + \underbrace{\frac{z_d^2}{2(b_d - a_d)}}_{\text{Convex}}$$

$$\Theta_r(z_r) = \frac{b_r^2}{2(b_r - a_r)} - \underbrace{\frac{b_r}{b_r - a_r} z_r}_{\text{Linear}} + \underbrace{\frac{z_r^2}{2(b_r - a_r)}}_{\text{Convex}}$$

$$\Lambda_d(z_d) = \frac{a_d^2}{2(b_d - a_d)} - \underbrace{\frac{a_d}{b_d - a_d} z_d}_{\text{Linear}} + \underbrace{\frac{z_d^2}{2(b_d - a_d)}}_{\text{Convex}}$$

$$\Lambda_r(z_r) = \frac{b_r^2}{2(b_r - a_r)} - \underbrace{\frac{b_r}{b_r - a_r} z_r}_{\text{Linear}} + \underbrace{\frac{z_r^2}{2(b_r - a_r)}}_{\text{Convex}}$$

$$\lambda_1 c - \lambda_1 w = 0$$

Linear Bilinear

$$\lambda_2 w - \lambda_2 p_d = 0$$

Bilinear Bilinear

$$\lambda_3 a_d - \lambda_3 z_d = 0$$

Linear Bilinear

$$\lambda_4 z_d - \lambda_4 b_d = 0$$

Bilinear Linear

$$\underbrace{\lambda_5 p_d^{\min}}_{\text{Linear}} - \lambda_5 p_d = 0 - \underbrace{\lambda_6 p_d^{\max}}_{\text{Linear}} = 0$$

$$\underbrace{\lambda_6 p_d}_{\text{Bilinear}}$$

$$\lambda_i \geq 0 \quad \forall i = 1, 2, 3, 4, m_r \geq 0, a_r \leq z_r, z_r \leq b_r, p_r \geq p_r^{\min}, p_r \leq p_r^{\max}$$

All constraints are linear

$$\underbrace{c \leq w, w \leq p_d, a_d \leq z_d, z_d \leq b_d, p_d \geq p_d^{\min}, p_d \leq p_d^{\max}}_{\text{All constraints are linear}}$$

It is worth nothing that in preprocessing step we considered each of $\Lambda_i(z_i)$ and $\Theta_i(z_i)$ as a decision variable and then defined them as four constraints.

Fortunately, the problem (12) doesn't have any general nonconvex terms and we don't need any α calculations. The nonconvex terms of

problem (12) are just of bilinear type and have customized convex lower bound that is stated in Theorem 1.

Theorem 1. The tightest convex lower bound of a bilinear term xy over the domain $[x^L, x^U] \times [y^L, y^U]$ is obtained by using a new variable ψ which changes every presence of xy in the problem and meets the following condition.

$$\psi = \max \{x^L y + y^L x - x^L y^L; x^U y + y^U x - x^U y^U\} \tag{13}$$

This lower bound can be converted to two linear inequality constraints in the problem:

$$\begin{aligned} \psi &\geq x^L y + y^L x - x^L y^L \\ \psi &\geq x^U y + y^U x - x^U y^U \end{aligned} \tag{14}$$

Proof. See Al-Khayyal and Falk (1983) or Adjiman et al. (1998).

To use theorem 1, we need to have the bounds of variables that are participating in the bilinear terms. We summarize the variable bounds in Table 2.

Table 2. Variable bounds in the α -BB algorithm

Variable	p_i	z_i	w	γ_r	$\Theta_r(z_r)$	$\Lambda_r(z_r)$	λ_i
Lower-bound	p_i^{min}	z_i^{min}	c	$a\delta - \alpha k p_r^{max} + \beta(p_d^{min} - p_r^{max})$	0	0	0
Upper-bound	p_i^{max}	z_i^{max}	p_{min}	$a\delta - \alpha k p_r^{min} + \beta(p_d^{max} - p_r^{min})$	A	A	$\bar{\lambda}_i$

where $A = \left(\frac{1}{b_r - a_r}\right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2}\right)$ and $p^{min} = \min\{p_r^{min}, p_d^{min}\}$.

Using **Theorem 1** and variable bounds of Table 2, we replace each bilinear nonconvex term of problem (12) with its equivalent new variable and add the necessary constraints to construct their valid convex under-estimator.

We define new variable $\Psi_1, \Psi_2, \dots, \Psi_{13}$ and their necessary conditions as follow:

$$p_r \gamma_r = \psi_1$$

$$\psi_1 \geq p_r^{\min} \gamma_r + (a\delta - \alpha k p_r^{\max} + \beta(p_d^{\min} - p_r^{\max})) p_r - (a\delta - \alpha k p_r^{\max} + \beta(p_d^{\min} - p_r^{\max})) p_r^{\min}$$

$$\psi_1 \geq p_r^{\max} \gamma_r + (a\delta - \alpha k p_r^{\min} + \beta(p_d^{\max} - p_r^{\min})) p_r - (a\delta - \alpha k p_r^{\min} + \beta(p_d^{\max} - p_r^{\min})) p_r^{\max}$$

$$w \gamma_r = \psi_2$$

$$\psi_2 \geq c \gamma_r + (a\delta - \alpha k p_r^{\max} + \beta(p_d^{\min} - p_r^{\max})) w - c(a\delta - \alpha k p_r^{\max} + \beta(p_d^{\min} - p_r^{\max}))$$

$$\psi_2 \geq p^{\min} \gamma_r + (a\delta - \alpha k p_r^{\min} + \beta(p_d^{\max} - p_r^{\min})) w - p^{\min}(a\delta - \alpha k p_r^{\min} + \beta(p_d^{\max} - p_r^{\min}))$$

$$p_r \Theta_r(z_r) = \psi_3$$

$$\psi_3 \geq p_r^{\min} \Theta_r(z_r)$$

$$\psi_3 \geq p_r^{\max} \Theta_r(z_r) + \left(\frac{p_r}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right) - \left(\frac{p_r^{\max}}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right)$$

$$w \Theta_r(z_r) = \psi_4$$

$$\psi_4 \geq c \Theta_r(z_r)$$

$$\psi_4 \geq p^{\min} \Theta_r(z_r) + \left(\frac{w}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right) - \left(\frac{p^{\min}}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right)$$

$$w \Lambda_r(z_r) = \psi_5$$

$$\psi_5 \geq c \Lambda_r(z_r)$$

$$\psi_5 \geq p^{\min} \Lambda_r(z_r) + \left(\frac{w}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right) - \left(\frac{p^{\min}}{b_r - a_r} \right) \left(\frac{a_r^2}{2} - a_r b_r + \frac{b_r^2}{2} \right)$$

$$p_d z_d = \psi_6$$

$$\psi_6 \geq p_d^{\min} z_d + a_d p_d - p_d^{\min} a_d$$

$$\psi_6 \geq p_d^{\max} z_d + b_d p_d - b_d p_d^{\max}$$

$$\lambda_1 w = \psi_7,$$

$$\psi_7 \geq c \lambda_1$$

$$\psi_7 \geq \bar{\lambda}_1 w + p^{\min} \lambda_1 - p^{\min} \bar{\lambda}_1$$

$$\lambda_2 w = \psi_8$$

$$\psi_8 \geq c \lambda_2$$

$$\psi_8 \geq \bar{\lambda}_2 w + p^{\min} \lambda_2 - \bar{\lambda}_2 p^{\min}$$

$$\begin{aligned} \lambda_2 p_d &= \psi_9 \\ \psi_9 &\geq p_d^{\min} \lambda_2 \\ \psi_9 &\geq \bar{\lambda}_2 p_d + p_d^{\max} \lambda_2 - \bar{\lambda}_2 p_d^{\max} \\ \lambda_3 z_d &= \psi_{10} \\ \psi_{10} &\geq a_d \lambda_3 \\ \psi_{10} &\geq \bar{\lambda}_3 z_d + b_d \lambda_3 - \bar{\lambda}_3 b_d \\ \lambda_4 z_d &= \psi_{11} \\ \psi_{11} &\geq a_d \lambda_4 \\ \psi_{11} &\geq \bar{\lambda}_4 z_d + b_d \lambda_4 - \bar{\lambda}_4 b_d \\ \lambda_5 p_d &= \psi_{12} \\ \psi_{12} &\geq p_d^{\min} \lambda_5 \\ \psi_{12} &\geq \bar{\lambda}_5 p_d + p_d^{\max} \lambda_5 - \bar{\lambda}_5 p_d^{\max} \\ \lambda_6 p_d &= \psi_{13} \\ \psi_{13} &\geq p_d^{\min} \lambda_6 \\ \psi_{13} &\geq \bar{\lambda}_6 p_d + p_d^{\max} \lambda_6 - \bar{\lambda}_6 p_d^{\max} \end{aligned}$$

The *relaxed single-level* problem (R) is the problem (12) that its bilinear terms are replaced by $\Psi_{i=1, \dots, 13}$ and also consists of necessary inequality constraints of these variables.

α -Branch and Bound (α -BB) algorithm steps

In this section we summarize the α -BB algorithm, as stated in Androulakis et al. (1995), to solve our problem. However, the following algorithm doesn't include α calculations and updates because we had just bilinear terms with customized lower bound.

STEP 1 – Solve initialization

1. Two tolerances ε_c and ε_f for convergence and feasibility are selected.
2. In this step, the iteration counter *Iter* takes the value of one.

3. The variable bounds for this first step $x^{L, Iter}$, $x^{U, Iter}$ are set to global ones x^L, x^U .
4. A lower and an upper bound (LBD, UBD) are initialized for the global minimum of (P).
5. An initial point $x^{c, Iter}$ is selected.

STEP 2 – Obtain a local solution for nonconvex NLP and upper bound updating

In this step, within the variable bounds x^L, x^U , the problem (P) is solved locally. We can use any commercial solvers such as MINOS, CONOPT, and KNITRO. If the solution f_{Local}^{Iter} of (P) is feasible with adopted tolerance ε_f , the UBD is updated as follow:

$$UBD = \min(UBD, f_{Local}^{Iter}) \quad (15)$$

STEP 3 – Dividing current bound rectangle

The rectangle bound $[x^{L, Iter}, x^{U, Iter}]$ is divided into the following two rectangles $r= 1, 2$ as follow:

$$r = 1: \begin{bmatrix} x_1^{L, Iter} & x_1^{U, Iter} \\ \vdots & \vdots \\ x_{l^{Iter}}^{L, Iter} & \frac{x_{l^{Iter}}^{L, Iter} + x_{l^{Iter}}^{U, Iter}}{2} \\ \vdots & \vdots \\ x_N^{L, Iter} & x_N^{U, Iter} \end{bmatrix}, r = 2: \begin{bmatrix} x_1^{L, Iter} & x_1^{U, Iter} \\ \vdots & \vdots \\ \frac{x_{l^{Iter}}^{L, Iter} + x_{l^{Iter}}^{U, Iter}}{2} & x_{l^{Iter}}^{U, Iter} \\ \vdots & \vdots \\ x_N^{L, Iter} & x_N^{U, Iter} \end{bmatrix} \quad (16)$$

where l^{Iter} indicates the variable with the longest side in the initial rectangle:

$$l^{Iter} = \arg \max_i (x_i^{U, Iter} - x_i^{L, Iter}) \quad (17)$$

STEP 4 - Solution of (R) inside both ranges $r = 1, 2$

In this step, inside both sub-rectangles ($r = 1, 2$), the convex problem (R) is solved. If a solution $l_{sol}^{r, Iter}$ is less than the UBD, then it is stored. Its solution point is named as $x_{sol}^{r, Iter}$.

STEP 5 - Update *Iter* and *LBD*

1. The iteration counter is increased by one, $Iter \leftarrow Iter + 1$.
2. *LBD* is updated to the minimum solution of previous iterations.
3. The selected solution is erased from the stored set.

$$LBD = l_{sol}^{r',Iter'} \text{ where } l_{sol}^{r',Iter'} = \min_{r,I} l_{sol}^{r,I} \quad r = 1, 2; I = 1, \dots, Iter - 1$$

STEP 6 - Update Current Point $x^{c, Iter}$ and Current Bounds $x^{L, Iter}, x^{U, Iter}$ on x

The current point is selected to be the solution point of the previously found minimum solution in STEP 6, $x^{c, Iter} = x_{sol}^{r',Iter'}$ and the current rectangle becomes the sub-rectangle containing the previously found solution.

$$[x^{L, Iter}, x^{U, Iter}] = \begin{cases} \begin{bmatrix} x_1^{L, Iter'} & x_1^{U, Iter'} \\ \vdots & \vdots \\ x_{Iter'}^{L, Iter'} & \frac{x_{Iter'}^{L, Iter'} + x_{Iter'}^{U, Iter'}}{2} \\ \vdots & \vdots \\ x_N^{L, Iter'} & x_N^{U, Iter'} \end{bmatrix} & \text{If } r' = 1 \\ \begin{bmatrix} x_1^{L, Iter'} & x_1^{U, Iter'} \\ \vdots & \vdots \\ \frac{x_{Iter'}^{L, Iter'} + x_{Iter'}^{U, Iter'}}{2} & x_{Iter'}^{U, Iter'} \\ \vdots & \vdots \\ x_N^{L, Iter'} & x_N^{U, Iter'} \end{bmatrix} & \text{If } r' = 2 \end{cases} \quad (18)$$

STEP 7 - Check for Convergence

If $(UBD - LBD) > \epsilon_c$, then return to STEP 2

Otherwise, we reached to the adopted convergence tolerance, ϵ_c .

The global minimum solution and solution point are: $f^* \leftarrow f^{c, Iter''}$
 $x^* \leftarrow x^{c, Iter''}$

where $iter'' = \arg \{f^{c,I} = UBD\}, I = 1, \dots, Iter$

We linked MATLAB R2015a with GAMS 24.5 to code the solution procedure. All of the computations have been performed on a

personal computer with AMD A10, 2.5 GHz and 8.00 GB RAM. The numerical results of the model are summarized in the next section.

Numerical results and managerial insights

In this section, we report numerical experiments to investigate the equilibrium point of a decentralized retailer-Stackelberg dual-channel supply chain.

The first set of experiments analyzes how the different channel prices and profits are influenced by customers' channel preference (a). In this experiment, we show that the online store may be beneficial to the traditional retailer. We obtain a Pareto-range of customers' channel preference that shows all supply chain members benefit from the online store.

The second experiment is focused on analyzing the Pareto-zone and the simultaneous impact of customers' channel preference (a) and relative price sensitivity (k) on it. Finally, in the last part we investigate the impact of model parameters on the Stackelberg-equilibrium.

The impact of customer's channel preference (a) on pricing strategies and profit functions

Consider a supply chain with $\delta = 10000$, $k = .45$, $\alpha = 60$, $v_i = s_i = 5$, $\beta = 10$, $\varepsilon_i \sim \text{Uniform } [0, 40]$. As it is shown in Figure 2-(1), with the increment of the value a , the price of physical retailer is increased. In situations when most customers in sales channel prefer to buy physically (high values of a), the new online store is not a serious threat to the retailer. In this case, the manufacturer is to appeal to customers' needs to have a price decrease.

As it is clear in Figure 2-(1), we have three different ranges for a :

- I. When $a \leq .04$, the online store is much more appealing to the customers. As a result, the physical retailer to attract more customers has no choice but to set his/her price equal to the wholesale price.
- II. When $.04 < a < .65$, almost both channels have their special customers. In this scenario, the retailer sets his/her price greater than wholesale price $p_r > w$ and as a result obtains an increasing profit margin. In this case, the competition between sales

channels is not so severe to force the manufacture to choose $p_d=w$.

- III. When $a \geq .65$ physical retailer is much more appealing to the customers and the manufacturer to obtain customers has no choice but to set $p_d=w$.

To compare the manufacturer and retailer’s profit in different cases DRS and SRS, we define profit differences by $\Delta\Pi_r = \Pi_r^{DRS} - \Pi_r^{SRS}$ and $\Delta\Pi_m = \Pi_m^{DRS} - \Pi_m^{SRS}$, respectively. As mentioned earlier, the Pareto-zone is a range of parameter a in which the manufacturer can open a direct online channel without any concerns about conflict due to the competition between his/her owned online store with the traditional channel. As it is shown in Figure 2-(2), the range of $a \geq .70727$ is a Pareto-zone because for this range we have $\Delta\Pi_m \geq 0$ and $\Delta\Pi_r \geq 0$.

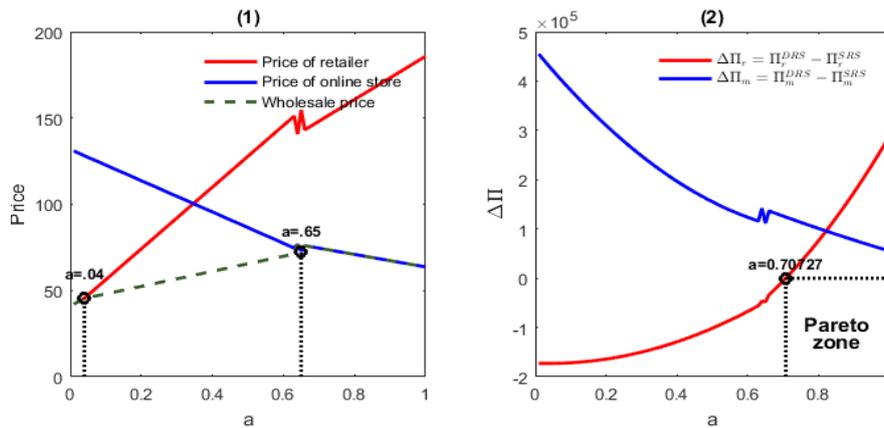


Fig. 2. A schematic presentation of Pareto-zone

Pareto-zone analysis

In a market with low sensitivity to the price of physical retailer and high sensitivity to the online store’s price, the manufacturer chooses low price for the online store to attract customers. Consequently, regarding the fact that always $w \leq p_d$, the retailer buys the product with lower price and earns more profit. Consider a supply chain with $\delta = 10000$, $k = .45$, $\alpha = 60$, $v_i = s_i = 5$, $\beta = 10$, $\varepsilon_i \sim \text{Uniform} [0, 40]$ to perform this analysis.

As it is clearly shown in Fig. 3, for ($k=.25$), there is a wide Pareto-zone. In this case, even when the parameter of a has a relatively low value, online store is beneficial to both of the manufacturer and the retailer. When the customers become more sensitive to retail price, for example $k=.5$, the value of a needs to be relatively high to have $\Delta\Pi_r \geq 0$ and it results in a smaller Pareto-zone. For very high values of k , the retailer cannot benefit from the new online store because in this situation the price of online store is much less than the retailer price and consequently the online store attracts most of the customers.

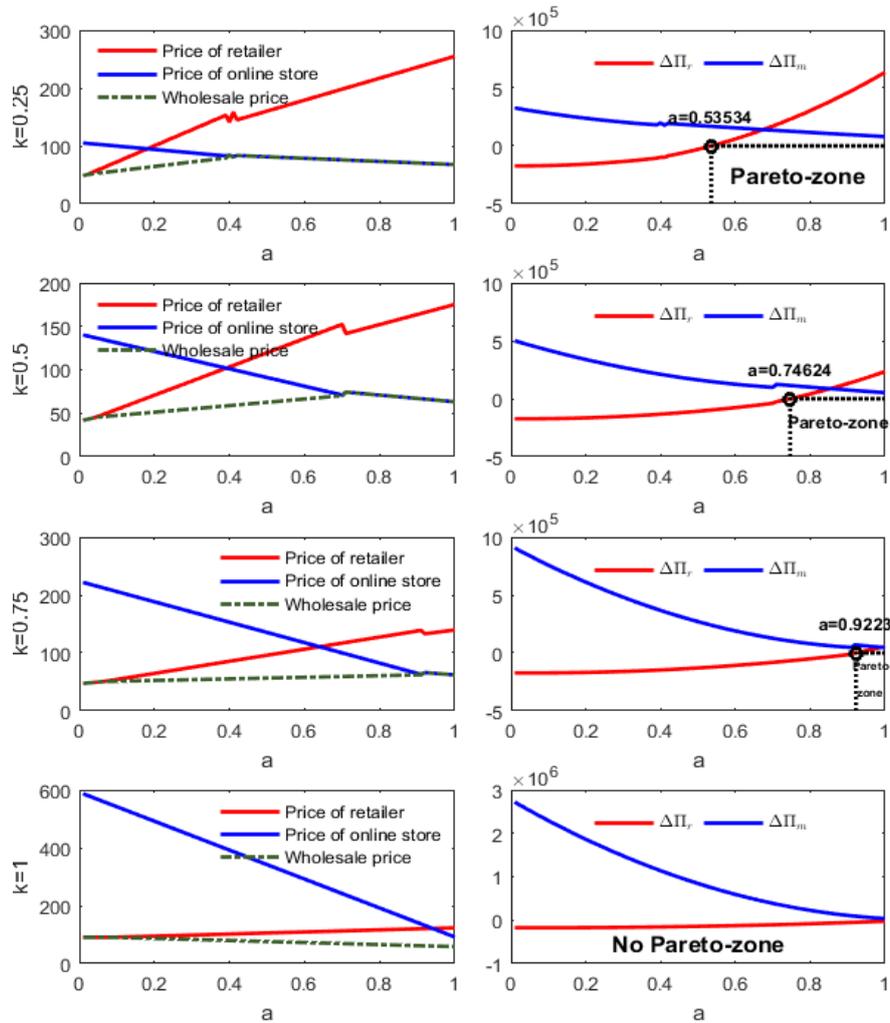


Fig. 3. Pareto-zone analysis

Managerial Insight 1. Stackelberg-equilibrium of the retailer Stackelberg dual-channel is concluded as follow:

- I. When the online store is much more appealing to the customers and the value of customers' channel preference parameter is very low, the retailer to interest customers should decrease its price. We assume that the game players are rational; then the

physical retailer would not cut down the price below the wholesale price. Even by setting $p_r=w$, the product is distributed among customers only through the online store.

- II. For an interval range of the customers' channel preference, both channels fairly have their customers and, thus, products are sold through both sales channels.
- III. When the physical store is much more appealing to the customers and the parameter a has high value, the manufacturer sets $p_d=w$ to interest customers to online buying. However, if the customers' channel preference for the physical store be very high, even by setting $p_d=w$ the online store will not have any customers.

Managerial Insight 2. Comparison of the profit function of the retailer for the scenarios of single retail Stackelberg and dual retail Stackelberg shows that in cases that the value of customers' channel preference is very low, by opening a new online store the profit of the retailer would be zero and he/she would not survive in the market. Then in this situation, the retailer has a better performance in the single channel sales network.

In summary, there are two different scenarios:

1. When the value of customers' channel preference parameter is very low, the retailer will exit from the market.
2. When the parameter a is upper than a threshold value, the retailer would benefit from the dual channel in case that parameter β be low. In this situation, the customer switches from the traditional retailer to the online store which is relatively low. As a result, the online store is not a serious competitor to the traditional retailer and the game between the manufacturer and the retailer will end in favor of the retailer.

Sensitivity analysis

In this part of the numerical analysis, we report the equilibrium results for three different examples in Table and clarify them as follow:

- I. Results of example 1 in Table 3 show that leader-retailer sets higher sale prices when the value of a goes up. The situation for the manufacturer is opposite. The manufacturer decreases the price of online store to interest more customers to online buying.

- II. Results of examples 1 and 2 of Table 3 show that with the increment of price sensitivity, the retailer has no choice but decreasing his/her price to stop losing customers. The manufacturer takes advantage of this situation and sets a higher wholesale and online store prices in order to increase his/her profit.
- III. Results of examples 1 and 3 of Table 3, which have different β values, reveal that for high values of β , a minor price difference between the two channels causes a great number of customer switches. Then in such a situation, the price of channels closes to each other.

As the results of examples in Table 3 show, when the online store is much more attractive to the customers ($a=6\%$ in example 1, $a=4\%$ in example 2 and $a=6\%$ in example 3), the retailer to interest more customers should lower his/her price. Even by setting $p_r=w$, the value of γr is zero and as a result the expected demand of retailer is $E[Dr]=E[\epsilon r]=20$. This expected demand is not enough to confront the cost of shortage inventory; then, the profit of the retailer is negative and the retailer would not survive in the supply chain. As the value of a increases, the physical retailer increases his/her price to take advantage of customers' channel preference. The situation for the online store is opposite. The online store cuts down his/her price to persuade the customers to buy online. When the physical retailer is much more attractive to customers (92% in example 1, 64% in example 2 and 100% in example 3), the manufacturer has no choice but to set $p_d^{RS}=w^{RS}$. However, sometimes even this policy would not bring any customer to the online store and we would have $\gamma d=0$.

When the competition pressure that is exerted through direct online channel is high (64% in example 1, 35% in example 2 and 61% in example 3), the retailer to allure more customers has to set his/her price lower than the online store's price.

Our numerical results match with results in Cai (2010). Cai (2010) investigates a manufacturer-Stackelberg scenario and shows the existence of a Pareto-zone, and in this paper we show that the same feature exists in a retailer-Stackelberg scenario.

Table 3. Equilibrium solution of retailer-Stackelberg game

Retailer-Stackelberg dualchannel example 1: $\delta= 10000, k= .75, \alpha= 60, \beta= 10, c=15, v_i= s_i=5$									
α	p_r	w	p_d	γ_r	γ_d	z_r	z_d	Π_r	Π_m
0.06	50.24623	50.24623	212.9804	0	4577.951	3.978851	31.50227	-90.049	908542.8
0.1	54.16109	51.06204	205.9246	80.38592	4393.496	4.832965	31.05264	172.4492	845002.9
0.2	64.62607	52.42154	188.2841	328.4077	3939.157	6.86103	29.92554	3988.075	697895.9
0.3	75.08478	53.77999	170.6403	576.7399	3484.841	8.322411	28.56542	12341.23	567486.4
0.4	85.54083	55.13907	152.993	825.1848	3030.583	9.42522	26.89115	25225.74	453785.5
0.5	95.9967	56.50034	135.3415	1073.596	2576.43	10.28641	24.77914	42633.89	356801.7
0.6	106.4546	57.86559	117.6833	1321.829	2122.463	10.97672	22.03124	64554.29	276544.4
0.64	110.6389	58.41334	110.6172	1421.031	1940.959	11.21571	20.68534	74582.04	249127.6
0.7	116.9172	59.23736	100.0133	1569.689	1668.84	11.54136	18.30794	90968.07	213026.7
0.8	127.388	60.61958	82.3189	1816.851	1215.907	12.01043	12.97361	121840.5	166271
0.9	137.873	62.01893	64.56644	2062.648	764.5693	12.40484	4.675812	157098.1	136321.7
0.91	138.9227	62.16019	62.78504	2087.103	719.6009	12.44081	3.583558	160859.6	134254.3
0.92	133.0209	65.9739	65.9739	2543.589	480.8616	14.09792	3.031502	171125.3	155813.7
1	139.5924	62.04702	62.04702	2942.89	0	15.31327	3.223362	228962.3	132719.5

Retailer-Stackelberg dualchannel example 2: $\delta= 10000, k= .45, \alpha= 60, \beta= 10, c=15, v_i= s_i=5$									
α	p_r	w	p_d	γ_r	γ_d	z_r	z_d	Π_r	Π_m
0.04	45.52339	45.43447	128.1696	0	4543.94	4.429128	27.38095	-88.8609	515999.4
0.1	56.27661	48.05699	122.7427	145.1928	4284.829	6.456997	25.96838	1149.459	468268
0.2	74.17481	52.42097	113.6916	392.448	3853.009	8.527793	23.31593	8597.975	396685.6
0.3	92.0617	56.78355	104.6343	640.0607	3421.34	9.791349	20.20399	22764.2	335092.3
0.35	101.0041	58.96581	100.1033	763.8832	3205.598	10.25446	18.43596	32361.9	308047.2
0.4	109.9467	61.14928	95.57042	887.6768	2989.939	10.64162	16.49931	43633.18	283505.6
0.5	127.835	65.52141	86.49735	1135.079	2558.964	11.25158	12.01237	71185.93	241939.8
0.6	145.7307	69.90313	77.41022	1382.067	2128.667	11.70912	6.462759	105395.4	210411.8
0.64	140.6572	76.74915	76.74915	1963.176	1706.358	15.49965	2.605892	126008.4	228689.2
0.7	148.1929	74.57466	74.57466	2262.609	1275.219	16.74159	2.681876	167266.3	212860.5
0.8	160.7481	70.94896	70.94896	2761.808	556.6759	18.55276	2.818928	248975.3	187729.8
0.9	173.2991	67.32166	67.32166	3261.149	0	20.10161	2.970812	346863.8	164166.6
1	185.8469	63.69304	63.69304	3760.596	0	21.4414	3.140061	460929.6	142174.6

Retailer-Stackelberg dual channel example 3: $\delta= 10000, k= .75, \alpha= 60, \beta= 30, c=15, v_i= s_i=5$									
α	p_r	w	p_d	γ_r	γ_d	z_r	z_d	Π_r	Π_m
0.06	70.02166	70.02166	153.8081	0	4579.287	2.855729	23.09018	-92.8594	635767.7
0.1	72.34526	70.13742	150.1746	79.34457	4412.499	3.373115	22.65022	89.20373	602984.6
0.2	78.73242	69.84988	141.0916	327.8165	4012.851	4.791269	21.61481	2855.766	526051.2
0.3	85.11755	69.56191	132.0067	576.3839	3613.227	5.996066	20.43677	8951.02	456227.8
0.4	91.50135	69.27385	122.9199	824.9949	3213.647	7.032314	19.08431	18372.48	393518.7
0.5	97.88437	68.98602	113.831	1073.603	2814.135	7.933068	17.51544	31117.27	337927.9
0.6	104.2671	68.6987	104.7397	1322.161	2414.724	8.72326	15.67353	47181.83	289459.1
0.61	104.9054	68.67001	103.8304	1347.012	2374.791	8.796978	15.47155	48970.71	285004
0.7	110.6499	68.41219	95.64524	1570.615	2015.461	9.42208	13.48025	66561.47	248116.1
0.8	117.0331	68.12676	86.54618	1818.901	1616.416	10.04458	10.82401	89249.71	213903.6
0.9	123.417	67.84263	77.44022	2066.928	1217.702	10.60283	7.540055	115237.1	186827.6
1	122.8807	72.02985	72.02985	2944.84	445.0791	11.96818	2.776627	150096.2	195056.6

Conclusion

We study situations when a retailer-Stackelberg dual-channel supply chain can improve the performance for the retailer as well as for the manufacturer, in comparison with a single retail-channel supply chain. In particular, we used a bi-level programming model to find the Stackelberg equilibrium point of wholesale and retail prices as well as order quantities for a retailer-dominant dual-channel supply chain. The results show that when the online store is much more appealing to the customers, the retailer to interest customers should decrease his/her price. For an interval range of the customers' channel preference, both channels fairly have their customers and, thus, products are sold

through both sales channels. At last, we show that when the physical store is much more appealing to the customers and the parameter a has high value, the manufacturer sets $p_d=w$ to interest customers to online buying.

The results also show that an online channel is not always detrimental to a retailer and there might be a range of customers' channel preference that the profit for both the retailer and manufacturer is improved; we call this range a Pareto-zone. Our results can provide efficient guidance for the manufacturer's decision on adopting a dual channel structure. At last, we also investigate the impact of the customers' channel preference on the equilibrium. This investigation shows that if customers' preference for the retailer is very low, the retailer will exit from the market and for a range of customers' channel preference, both the manufacturer and the retailer will compete to attract more customers.

In this paper we investigated a situation with one manufacturer and one retailer. A potential extension of this paper is to examine the interaction among multiple retailers and manufacturers. Another interesting study might be incorporating risk in the study and considering the retailer as a risk-averse member.

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Appendix 1

Average salvaged inventory is obtained as follow:

$$\begin{aligned}\Lambda_i(q_i) &= E(q - D)^+ = E(\gamma_i + z_i - \gamma_i - \varepsilon_i)^+ \\ \Rightarrow \Lambda_i(z_i) &= \int_{a_i}^{z_i} (z_i - \varepsilon_i) f(\varepsilon_i) d\varepsilon_i = \int_{a_i}^{z_i} z_i f(\varepsilon_i) d\varepsilon_i - \int_{a_i}^{z_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i\end{aligned}$$

Now to solve second integral, we use integration by parts as follow:

$$\begin{aligned}u &= \varepsilon_i, dv = f(\varepsilon_i) d\varepsilon_i \\ \Rightarrow \int_{a_i}^{z_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i &= \varepsilon_i F(\varepsilon_i) \Big|_{a_i}^{z_i} - \int_{a_i}^{z_i} F(\varepsilon_i) d\varepsilon_i = z_i F(z_i) - \int_{a_i}^{z_i} F(\varepsilon_i) d\varepsilon_i\end{aligned}$$

So we have:

$$\Lambda_i(z_i) = z_i F(z_i) - z_i F(z_i) + \int_{a_i}^{z_i} F(\varepsilon_i) d\varepsilon_i = \int_{a_i}^{z_i} F(\varepsilon_i) d\varepsilon_i$$

Average shortage inventory is obtained as follow:

$$\begin{aligned}\Theta_i(q_i) &= E(D - q)^+ = E(\gamma_i + \varepsilon_i - \gamma_i - z_i)^+ \\ \Rightarrow \Theta_i(z_i) &= \int_{z_i}^{b_i} (\varepsilon_i - z_i) f(\varepsilon_i) d\varepsilon_i = \int_{z_i}^{b_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i - z_i (1 - F(z_i))\end{aligned}$$

Using integration by parts for first integral, we would have:

$$\begin{aligned}\int_{z_i}^{b_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i &= \int_{a_i}^{b_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i - \int_{a_i}^{z_i} \varepsilon_i f(\varepsilon_i) d\varepsilon_i = \mu_{\varepsilon_i} - z_i F(z_i) - \int_{a_i}^{z_i} F(\varepsilon_i) d\varepsilon_i \\ \Rightarrow \Theta_i(z_i) &= \mu_{\varepsilon_i} - z_i F(z_i) - \Lambda_i(z_i) - z_i (1 - F(z_i)) = \mu_{\varepsilon_i} - z_i + \Lambda_i(z_i)\end{aligned}$$