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# Solving a Two-Period Cooperative Advertising Problem Using Dynamic Programming

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## Abstract
Cooperative advertising is a cost-sharing mechanism in which a part of retailers' advertising investments are financed by the manufacturers. In recent years, investment among advertising options has become a difficult marketing issue. In this paper, the cooperative advertising problem with advertising options is investigated in a two-period horizon in which the market share in the second period depends on the decisions made in the first period. The problem is solved for two cases of the absence and presence of cooperative advertising contract, and the results are compared. The solution to the problem is presented using the concepts of the Nash equilibrium, Stackelberg game, stochastic games, and dynamic programming. The computational results using numerical examples show that if the cooperative advertising contract is offered in the win-win condition, the players' profit will increase significantly.

## Keywords
Cooperative advertising, Game theory, Stochastic games, Advertising options.

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## Introduction

Cooperative advertising is a cost-sharing mechanism and an advertising scheme adopted by manufacturers to influence retailers' behavior (Berger, 1972). In a cooperative advertising contract, manufacturer's participation is often expressed as a percentage of retailer's advertising costs (Bergen & John, 1997). Cooperative advertising has been highly promoted in IBM (Brennan, 1988), Apple (Clark, 2000), and an online version in automotive and durable goods (Lieb, 2012).

A "two-period" phenomenon often occurs in product sales (Zhou et al., 2015). For example, Gucci and Prada sell their products at a regular price in the 1st period and then at a discounted price when a newer version is launched (Xu et al. 2019). Furthermore, a "two-period" phenomenon often occurs in the consumer-electronics industry when a new style of a product is launched. The first period is from the time a new product (e.g. Apple iPhone 6) is launched into the market to the time a newer-generation product appears. The second period is from the time that a newer-generation product (e.g. Apple iPhone 6S) appears in the market to the time this product exits the market (He et al. 2019). The supply chain members must determine their strategies that will be applied in two-period horizon.

In this paper, the cooperative advertising with two advertising options is investigated in a two- period horizon, in which the market share in the 2nd period depends on the decisions made in the 1st period. We consider two cases of absence and presence of cooperative advertising contract for the problem. The solution to the problem is presented using the concepts of the Nash equilibrium (Nash, 1950), Stackelberg game (Stackelberg, 1934), stochastic games, and dynamic programming. Using numerical examples, the conditions for which cooperative advertising can benefit all players are identified. The paper extended the previous works on some directions: (a) cooperative advertising problem with two local advertising options is considered; (b) the problem is investigated in two periods using the concepts of stochastic games and dynamic programming; and (c) two cases of the absence and presence of cooperative advertising are compared in order to identify win-win conditions.

In the cooperative advertising literature, some researchers study monopoly, duopoly, and oligopoly in the downstream echelon.

Examples include Aust and Buscher (2014a), Hong et al. (2015), and Zhao et al. (2016) for monopoly, Alaei et al. (2014), Zhang and Zhong (2011), Aust and Buscher (2014b), and Giri and Sharma (2014) for duopoly, Chutani and Sethi (2012) and Jørgensen and Zaccour (2003) for oligopoly setting.

In recent years, investment among advertising options has become a difficult marketing issue. Frison et al. (2014) state that different media such as television, radio, magazines, newspapers, billboards, cinema, etc. can differ widely in their short- and/or long-run effectiveness. It seems that neglecting the effectiveness of different media in the problem may lead to sub-optimal solutions.

We compare the presence and absence of the cooperative advertising in order to identify the win-win condition. Some studies also investigate the sufficient incentives of players to accept the cooperative advertising contract. Jorgensen et al. (2001) show that a cooperative advertising contract is Pareto improving. Yang et al. (2013) identify the necessary conditions for perfect channel coordination when the retailer has fairness concerns. Chen (2011) and Giri and Bardhan (2014) compare the presence and absence of the cooperative advertising contract, and propose a channel rebate mechanism to get a win-win condition. Giri and Sharma (2014) and Karray and Hassanzadeh Amin (2015) also study the presence and absence of cooperative advertising contract. However, they do not identify the win-win condition.

There have been limited research studies on cooperative advertising considering two-period supply chain. For example, Xiao et al. (2010) study this problem in a supply chain with a retailer and a manufacturer in order to identify the win-win condition. He et al. (2014) also propose a coordination contract in a fashion, and textiles supply chain to investigate a two-period cooperative advertising problem. Martín-Herrán and Sigué (2017) investigate this problem in a retailer-manufacturer supply chain for determining their advertising and pricing strategies. Karray et al. (2017) also study this problem in a channel with two competing manufacturers in order to determine their advertising strategies. He et al. (2019) investigate a two-way subsidy contract for supply chain coordination considering the competition between two-generation products. Xu et al. (2019) consider retail competition and propose a two-way subsidy contract to achieve the channel coordination under the decentralized scenario.

## Description of the Model

Assume a supply chain, including a manufacturer and two competing retailers facing two advertising options for local advertising. The market demand of retailers depends on their local advertising investments, the national advertising investment of the manufacturer, and the selected pair of advertising options.

The following notation will be used in the paper. We use *i* and *j* as indices of local advertising options, in which *(i,j)* is the pair of local advertising options where the retailers 1 and 2 select *i* and *j*, respectively. $k_i$ and $k_m$ denote the efficiency of the local advertising option *i* and national advertising on the market demand, respectively. The cost associated with the local advertising option *i* and national advertising are denoted by $c_i$ and *C*, respectively. $\rho_1$, $\rho_2$ and $\rho_m$ denote the profit margin of the retailer 1, retailer 2 and manufacturer, respectively. $\eta_{ij}$ is the retailer 1's market share for pair *(i,j)*. The corresponding profit functions of the supply chain members are denoted by $\Pi_{1|(i,j)}$, $\Pi_{2|(i,j)}$ and $\Pi_{m|(i,j)}$. The decision variables of each player must be determined by maximizing their profits, and are introduced as follows.

| | |
|---|---|
| A | Manufacturer's national advertising level |
| $a_{i\|(i,j)}$ | The retailer 1's local advertising level in advertising option i for pair (i,j) |
| $b_{j\|(i,j)}$ | The retailer 2's local advertising level in advertising option j for pair (i,j) |
| $t_1^{ij}$ | The participation rate of manufacturer in retailer 1's advertising for pair (i,j) |
| $t_2^{ij}$ | The participation rate of manufacturer in retailer 2's advertising for pair (i,j) |

### The Single-Period Problem

First, the model is formulated for single-period mode. This model is important because the two-period problem can be considered as two sub-problems of the first and second periods.

We apply a sale response function *h(a, b, A)* based on square roots for modeling the saturation effect (SeyedEsfahani et al., 2011; Alaei et al. 2014; Aust and Buscher, 2014b; Alaei & Setak, 2016). Additionally, we assume that each retailer's local advertising will also influence his competitor's demand. This approach is followed by other researchers (Karray and Zaccour, 2007; Aust & Buscher, 2014b). This assumption can be applicable to the firms that offer after-sale services such as the warranty in the consumer-electronics industry that both sell the same product (e.g. Apple iPhone 6). The following sales response function proposed by Aust and Buscher (2014b) is used to model advertising effect, which is valid for each retailer:

$$h(\mathrm{a},\mathrm{b},\mathrm{A}) = k_r\sqrt{a} + k_r\sqrt{b} + k_m\sqrt{A} \tag{1}$$

where $k_m$ and $k_r$ are the effectiveness of the national and local advertising investments, respectively, and $a$, $b$, and $A$ denote the advertising levels of the retailer 1, retailer 2 and manufacturer, respectively.

We are going to extend the sale response function in (1) to a case where there exist two local advertising options available to the retailers. As stated before, different media such as television, radio, etc. can differ widely in their short- and/or long-run effectiveness (Frison et al. 2014). Neglecting the effectiveness of different media in the problem may lead to sub-optimal solutions.

Let $\eta$ be the market share of retailer 1. The parameter $\eta$ depends on retailers' behavior and indicates the mutual effectiveness of the selected advertising options. Now assume that the retailer 1 chooses option $i\in\{1,2\}$ with the effectiveness parameter $k_i$, and the retailer 2 selects option $j\in\{1,2\}$ with the effectiveness parameter $k_j$. Combining these characteristics, the market demand for retailers is obtained as following.

$$D_1(\mathrm{a}_i,\mathrm{b}_j,\mathrm{A},\eta) = \eta\left(k_i\sqrt{a_i} + k_j\sqrt{b}_j + k_m\sqrt{A}\right) \tag{2}$$

$$D_2(\mathrm{a}_i,\mathrm{b}_j,\mathrm{A},\eta) = (1-\eta)\left(k_i\sqrt{a_i} + k_j\sqrt{b}_j + k_m\sqrt{A}\right) \tag{3}$$

Note that the manufacturer's total demand will be $D = D_1 + D_2$. Considering the demand functions in (2) and (3), the profit functions of the manufacturer, retailer 1, and retailer 2 can be formulated as following, respectively.

$$\Pi_{m|(i,j)} = \rho_m\left(\mathrm{k}_i\sqrt{a_{i|(i,j)}} + \mathrm{k}_j\sqrt{b_{j|(i,j)}} + \mathrm{k}_m\sqrt{A}\right) - t_1^{ij}c_i a_{i|(i,j)} - t_2^{ij}c_j b_{j|(i,j)} - CA \tag{4}$$

$$\Pi_{1|(i,j)} = \rho_1\eta_{ij}\left(\mathrm{k}_i\sqrt{a_{i|(i,j)}} + \mathrm{k}_j\sqrt{b_{j|(i,j)}} + \mathrm{k}_m\sqrt{A}\right) - (1-t_1^{ij})c_i a_{i|(i,j)} \tag{5}$$

$$\Pi_{2|(i,j)} = \rho_2\left(1-\eta_{ij}\right)\left(\mathrm{k}_i\sqrt{a_{i|(i,j)}} + \mathrm{k}_j\sqrt{b_{j|(i,j)}} + \mathrm{k}_m\sqrt{A}\right) - (1-t_2^{ij})c_j b_{j|(i,j)} \tag{6}$$

The expressions in the parentheses denote the market demand function that is used by Aust and Buscher (2014b) for retailers' duopoly. In equations (4-6), the first terms represent the profit from the sale. The other terms are related to the local and national advertising costs. Equations (4-6) are the players' profit function in the presence of the contract. The corresponding profits in the absence of cooperative advertising are determined if the participation rates ($t_1^{ij}$ and $t_2^{ij}$) set to be zero. We assume $\eta$=0.5, either for the pair of advertising options (1,1) or (2,2). This assumption is completely consistent with relation (1) proposed by Aust and Buscher (2014b). When both retailers choose the same advertising option, it is as if only one option is available. In this case, each retailer's total demand is equal to the equation (1) and the share of each from the total demand $D=D_1+D_2$ is 0.5. Now consider that they choose a strategy with different advertising options, i.e. (1,2) or (2,1). Without loss of generality, consider that the second option is more effective than the first one, i.e. $k_2 > k_1$. It is reasonable that the retailer who chooses the second option attracts more customers than the retailer who chooses the first one. So, in these two strategies, we assume that the share of the retailer who chooses the first option is $\eta$, where $\eta$<0.5. Finally, the game $G(\eta)$ between retailers can be illustrated by Figure 1.

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_{1\|(1,1)}(0.5), \Pi_{2\|(1,1)}(0.5)\right)$ | $\left(\Pi_{1\|(1,2)}(\eta), \Pi_{2\|(1,2)}(1-\eta)\right)$ |
| 2 | $\left(\Pi_{1\|(2,1)}(1-\eta), \Pi_{2\|(2,1)}(\eta)\right)$ | $\left(\Pi_{1\|(2,2)}(0.5), \Pi_{2\|(2,2)}(0.5)\right)$ |

**Fig. 1. Representation of G(η), the η-Specific Game**

**The Two-Period Problem**

In this case, the assumptions of the single-period problem hold and each player aims to optimize the aggregate profit of two periods. Moreover, the strategy chosen in the 1$^{st}$ period is assumed to affect the retailers' credibility. For example, if the retailer 1 chooses the less effective advertising option in the 1$^{st}$ period (i.e. the pair (1,2)), it will have less market share in that period (i.e. $\eta < 0.5$), and if the same trend continues in the 2$^{nd}$ period, its market share will be less than before (i.e. $\eta' < \eta < 0.5$). In this case, $G(\eta')$ will be played in the 2$^{nd}$ period. Conversely, $G(\eta'')$ where $\eta''>\eta$, will be played in the 2$^{nd}$ period if the retailer 1 chooses the more effective advertising option in the 1$^{st}$ period (i.e. the pair (2,1)). Finally, if in the 1$^{st}$ period, retailers choose the same advertising options, then $G(\eta)$ will be played in the 2$^{nd}$ period.

(P$_2$)

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ | $\left(\Pi_1^2(\eta),\Pi_2^2(1-\eta)\right)$ |
| 2 | $\left(\Pi_1^2(1-\eta),\Pi_2^2(\eta)\right)$ | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ |

(P$_3$)

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ | $\left(\Pi_1^2(\eta'),\Pi_2^2(1-\eta')\right)$ |
| 2 | $\left(\Pi_1^2(1-\eta'),\Pi_2^2(\eta')\right)$ | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ |

(P$_1$)

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_1^1(0.5),\Pi_2^1(0.5)\right)$ | $\left(\Pi_1^1(\eta),\Pi_2^1(1-\eta)\right)$ |
| 2 | $\left(\Pi_1^1(1-\eta),\Pi_2^1(\eta)\right)$ | $\left(\Pi_1^1(0.5),\Pi_2^1(0.5)\right)$ |

(P$_4$)

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ | $\left(\Pi_1^2(\eta''),\Pi_2^2(1-\eta'')\right)$ |
| 2 | $\left(\Pi_1^2(1-\eta''),\Pi_2^2(\eta'')\right)$ | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ |

(P$_5$)

| | 1 | 2 |
|---|---|---|
| 1 | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ | $\left(\Pi_1^2(\eta),\Pi_2^2(1-\eta)\right)$ |
| 2 | $\left(\Pi_1^2(1-\eta),\Pi_2^2(\eta)\right)$ | $\left(\Pi_1^2(0.5),\Pi_2^2(0.5)\right)$ |

**Fig. 2. Games Representation in Two Periods**

So according to the above description, a stochastic game will be played in two periods. Stochastic games are the generalization of the iterative games that, at each stage, players play a game that is dependent on the game and the decisions of the previous stage. Figure 2 shows this situation where the retailers play $P_1$ in the 1$^{st}$ period, then play one of the games $P_2$, $P_3$, $P_4$, or $P_5$ in the 2$^{nd}$ period.

## Solution Approach

Game theory is the most commonly used methodology for analyzing the issue of cooperative advertising (Jørgensen & Zaccour, 2014). Since none of the retailers precede the other in decision-making, the Nash equilibrium can be used to determine their strategies (Nash, 1950). On the other hand, the manufacturer has a certain privilege that allows him to move first. The manufacturer and retailers act as the leader and followers of the Stackelberg game (Stackelberg, 1934).

### Single-Period Problem

The solution can be obtained using Nash and Stackelberg game. Proposition 1 and Proposition 2 determine the equilibrium solution in the presence and absence of the cooperative advertising contract, respectively.

**Proposition 1**. In the presence of contract, for any pair of local advertising options $(i, j)$, the equilibrium solution is as follows:

$$t_1^{ij} = \frac{2\rho_m - \rho_1\eta_{ij}}{2\rho_m + \rho_1\eta_{ij}}$$

$$t_2^{ij} = \frac{2\rho_m - \rho_2\left(1-\eta_{ij}\right)}{2\rho_m + \rho_2\left(1-\eta_{ij}\right)}$$

$$A = \left(\frac{\rho_m \mathrm{k}_m}{2C}\right)^2$$

$$a_{i|(i,j)} = \left[\frac{\mathrm{k}_i}{4c_i}\left(2\rho_m + \rho_1\eta_{ij}\right)\right]^2$$

$$b_{j|(i,j)} = \left[\frac{\mathrm{k}_j}{4c_j}\left(2\rho_m + \rho_2\left(1-\eta_{ij}\right)\right)\right]^2$$

**Proof**: The solution can be determined by backward induction. In the first stage, the retailers' best response function is obtained by simultaneously letting the first derivative of Equation (5) with respect to variables $a_{i|(i,j)}$ and that of Equation (6) with respect to variables $b_{j|(i,j)}$ to zero. Then, substituting the best responses into the profit function of the manufacturer in Equation (4), and solving the necessary conditions for the optimality of this function with respect to variables $A$, $t_1^{ij}$ and $t_2^{ij}$ leads to the equilibrium solution.

**Proposition 2**. In the absence of contract, for any pair of local advertising options $(i, j)$, the equilibrium solution is as follows:

$$A = \left(\frac{\rho_m \mathrm{k}_m}{2C}\right)^2$$

$$a_{i|(i,j)} = \left(\frac{\rho_1 \eta_{ij} \mathrm{k}_i}{2c_i}\right)^2$$

$$b_{j|(i,j)} = \left(\frac{\rho_2 \left(1-\eta_{ij}\right) \mathrm{k}_j}{2c_j}\right)^2$$

**Proof**. It can be proved similar to Proposition 1.

Algorithm for solving the single-period problem is as follows. Note that the algorithm can be used for both cases of the presence and absence of cooperative advertising contract.

**Algorithm 1**

*1- Determine the optimal value of decision variables in the presence (absence) of cooperative advertising according to* **Proposition 1** *(***Proposition 2***) for any pair (i,j).*

*2- Substitute the optimal values into equations (4-6) and calculate the optimal profits of the players for any pair (i,j) of local advertising options.*

*3- Form the payoff matrix of the game, G(η), as the game in* **Figure 1**.

*4- Determine the equilibrium pair of advertising options which is the Nash equilibrium of G(η).*

*5- Report the corresponding equilibrium decision variables computed in step 1.*

**Solution to the Two-Period Problem**

The problem-solving approach for two-period problem is dynamic programming with the following characteristics. **Stage**: decision making period ($p$=1, 2); **State**: the market share $\eta$; and **Decision variables**: advertising options and investments. In the following, each player's problem in the two-period case is introduced. It should be noted that the index denoting advertising options has been removed to make the formula simpler. The manufacturer's problem is as following:

$$\begin{aligned} \max_{A,t} \quad & J_M(A,t,a,b) = \Pi_M^1 + \Pi_M^2 \\ s.t. \quad & A = (\mathrm{A}^1, \mathrm{A}^2) \geq 0 \\ & 0 \leq \mathrm{t} = \left(t_1^1, t_2^1, t_1^2, t_2^2\right) \leq 1 \end{aligned} \qquad (7)$$

where supper-scripts represent the decision periods. Similarly, for retailers 1 and 2, the total profit function is defined as following, respectively.

$$\begin{aligned} &\max_{a} \quad J_1(A,t,a,b) = \Pi_1^1 + \Pi_1^2 \\ &s.t. \quad a = (a^1, a^2) \geq 0 \end{aligned} \tag{8}$$

$$\begin{aligned} &\max_{b} \quad J_2(A,t,a,b) = \Pi_2^1 + \Pi_2^2 \\ &s.t. \quad b = (\mathrm{b}^1, \mathrm{b}^2) \geq 0 \end{aligned} \tag{9}$$

It should be noted that due to the dependence of the second period' game on decisions of the first period, the investment amounts and participation rates are determined similar to the single-period problem. What is changed in the two-period version is that determining the equilibrium pair of options is done in both decision-making periods. The solution to the problem is the backward procedure that is described in the following.

**Stage 2 (the second period)**

With regard to the game played in the first period and the equilibrium pair of options, the market share is known at the beginning of this period. Assuming the state of the problem as $s=\eta_0$, the problem is equivalent to $G(\eta_0)$, which is presented in equation (10).

Solving $G(\eta_0)$ for any state of the problem, the equilibrium pair of options are obtained. It is assumed that the players' profit in the second period are denoted by $\bar{\Pi}_M(\eta_0), \bar{\Pi}_1(\eta_0)$ and $\bar{\Pi}_2(1-\eta_0)$.

$$G(\eta_0) \equiv \begin{cases} \max\limits_{A^2, t_1^2, t_2^2} \quad \Pi_M^2(\eta_0, \mathrm{a}^2, \mathrm{b}^2, \mathrm{A}^2, t_1^2, t_2^2) \\ s.t. \\ \mathrm{A}^2 \geq 0 \\ 0 \leq t_1^2, t_2^2 \leq 1 \\ a^2 = \arg\max\limits_{a^2} \quad \Pi_1^2(\eta_0, \mathrm{a}^2, \mathrm{b}^2, \mathrm{A}^2, t_1^2) \\ b^2 = \arg\max\limits_{b^2} \quad \Pi_2^2(1-\eta_0, \mathrm{a}^2, \mathrm{b}^2, \mathrm{A}^2, t_2^2) \end{cases} \tag{10}$$

**Stage 1 (the first period)**

At this stage, the advertising investments and participation rates can be calculated independently for each pair of options. Equations (11-13) introduce the optimization problems of the supply chain members.

$$\begin{aligned} &\max_{a^1} \quad \Pi_1^1 + \bar{\Pi}_1^2(\eta_0 \mid (\mathrm{i,j})) \\ &s.t. \quad a^1 \geq 0 \end{aligned} \tag{11}$$

$$\begin{aligned} &\max_{b^1} \quad \Pi_2^1 + \bar{\Pi}_2^2(1-\eta_0 \mid (\mathrm{i,j})) \\ &s.t. \quad b^1 \geq 0 \end{aligned} \tag{12}$$

$$\begin{aligned} &\max_{A^1, t_1^1, t_2^1 \mid (\mathrm{i,j})} \quad \Pi_M^1 + \bar{\Pi}_M^2(\eta_0 \mid (\mathrm{i,j})) \\ &s.t. \quad \mathrm{A}^1 \geq 0 \\ &\qquad 0 \leq t_1^1, t_2^1 \leq 1 \\ &\qquad a^1 = \max_{a^1} \quad \Pi_1^1 + \bar{\Pi}_1^2(\eta_0 \mid (\mathrm{i,j})) \\ &\qquad b^1 = \max_{b^1} \quad \Pi_2^1 + \bar{\Pi}_2^2(1-\eta_0 \mid (\mathrm{i,j})) \end{aligned} \tag{13}$$

The payoff table in Stage 1 is shown in Figure 3 in which the equilibrium pair of options can be determined using Nash equilibrium.

| | R | TV |
|---|---|---|
| R | $\begin{pmatrix} \Pi_1^1(0.5) + \bar{\Pi}_1^2(0.5), \\ \Pi_2^1(0.5) + \bar{\Pi}_2^2(0.5) \end{pmatrix}$ | $\begin{pmatrix} \Pi_1^1(\eta) + \bar{\Pi}_1^2(\eta_0 \mid (1,2)), \\ \Pi_2^1(1-\eta) + \bar{\Pi}_2^2(1-\eta_0 \mid (1,2)) \end{pmatrix}$ |
| TV | $\begin{pmatrix} \Pi_1^1(1-\eta) + \bar{\Pi}_1^2(\eta_0 \mid (2,1)), \\ \Pi_2^1(\eta) + \bar{\Pi}_2^2(1-\eta_0 \mid (2,1)) \end{pmatrix}$ | $\begin{pmatrix} \Pi_1^1(0.5) + \bar{\Pi}_1^2(0.5), \\ \Pi_2^1(0.5) + \bar{\Pi}_2^2(0.5) \end{pmatrix}$ |

**Fig. 3. Payoff Table of Stage 1, G($\eta_0$)**

Algorithm for solving the two-period problem is as follows. Note that the algorithm can be used for both cases of the presence and absence of cooperative advertising contract.

**Algorithm 2**
*1- Use* **Algorithm 1** *to solve $G(\eta_0)$ for any state of the problem ($\eta_0 = \eta, \eta', \eta''$)*
*2- Calculate the equilibrium profit of the players for step 1*
*3- Form the payoff matrix of $G(\eta_0)$ as the game in* **Figure 3**
*4- Determine the equilibrium pair of advertising options which is the Nash equilibrium of $G(\eta_0)$*
*5- Report the corresponding equilibrium decision variables computed in step 1.*

## Numerical Study

### Example Data

In this section, a two-period example with market configuration according to Table 1 is considered. There exist two local advertising options to retailers: television (T) and radio (R). Besides, the manufacturer uses television for national advertising. The goal is to determine the players' advertising decisions in both periods. Each player aims to optimize his profit at two periods. It should be noted that the television advertising has different cost and efficiency in the national and local advertising, as these parameters depend on advertising time and television channel. The retailers play $G(\eta)$ in the first period, and then play another game depending on the equilibrium solution of the first period. Considering the assumptions described in the previous section, we assume $\eta=0.43$, $\eta'=0.41$ and $\eta''=0.45$.

**Table 1. Market Configuration**

| Parameter | First period | Second period |
|---|---|---|
| $\rho_m$ | 15 | 15 |
| $\rho_1$ | 10 | 10 |
| $\rho_2$ | 12 | 12 |
| $k_m$ | 100 | 100 |
| $k_1$ | 50 | 50 |
| $k_2$ | 60 | 70 |
| $C$ | 250 | 250 |
| $c_1$ | 20 | 20 |
| $c_2$ | 40 | 60 |

### Absence of Cooperative Advertising Contract

As described before, we can solve the problem using dynamic programming: periods as the stages and the market shares as the states of the problem. In the following, the solution is presented using the backward procedure.

- **Stage 2**

The results regarding the states *G(0.41)*, *G(0.43)* and *G(0.45)* are depicted in Figure 4. The underlined payoffs for each state show the Nash equilibrium. Table 2 summarizes the results of the second period.

- **Stage 1**

In the first period, the state is $\eta=0.43$. Besides the retailers' profit in the first period, the equilibrium payoff of the second stage must be considered. According to the assumptions of the problem, the choice of strategies (R, R), (R, TV), (TV, R) and (TV, TV) in the 1st period, respectively, lead to the game *G(0.43)*, *G(0.41)*, *G(0.45)* and *G(0.43)* in the 2nd period. The payoff table of the 1st period is shown in Figure 5-(a), which is the summation of the first-period payoff and the equilibrium payoff of the corresponding game in the second stage. The equivalent payoff matrix is shown in Figure 5-(b).

*G (0.41)*

| | R | | TV | |
|---|---|---|---|---|
| R | 4156 | 4800 | 2941 | 4962 |
| TV | 4295 | 3418 | **3235** | **3760** |

*G (0.43)*

| | R | | TV | |
|---|---|---|---|---|
| R | 4156 | 4800 | 3069 | 4845 |
| TV | 4212 | 3581 | **3235** | **3760** |

*G (0.45)*

| | R | | TV | |
|---|---|---|---|---|
| R | **4156** | **4800** | 3296 | 4726 |
| TV | 4124 | 3744 | 3235 | 3760 |

**Fig. 4. The Equilibrium of G(0.41), G(0.43) and G(0.45) in the Absence of Contract**

**Table 2 The Result of Stage 2 (second period) in the Absence of Contract**

| State ($\eta$) | Equilibrium solution | Equilibrium payoff |
|---|---|---|
| 0.41 | (TV,TV) | (3235, 3760) |
| 0.43 | (TV,TV) | (3235, 3760) |
| 0.45 | (R,R) | (4156, 4800) |

a)

| | R | | TV | |
|---|---|---|---|---|
| R | 4156 + **3235** | 4800 + **3760** | 3191 + **3235** | 4943 + **3760** |
| TV | 4279 + **4156** | 3704 + **4800** | 3413 + **3235** | 3960 + **3760** |

↓

b)

| | R | | TV | |
|---|---|---|---|---|
| R | 7391 | 8560 | 6426 | 8703 |
| TV | **8435** | **8504** | 6648 | 7720 |

**Fig. 5. (a) Payoff Table of Stage 1 in the Absence of Contract; (b) the Equivalent Payoff Table**

The Nash equilibrium of the game in Figure 5-(b) is shown using underline. Choosing (TV, R) in the first period will result in playing *G(0.45)* in the second period and the equilibrium will be (R, R) according to Table 2. Therefore, the best strategy for retailers in the absence of cooperative advertising is playing (TV, R) and (R, R) in the first and second periods, respectively.

The decision variables, including the retailers' advertising levels at two periods are obtained as following: $a^1$=18.27, $a^2$=39.06, $b^1$=41.6 and $b^2$=56.25.

## Presence of Cooperative Advertising Contract

In the presence of cooperative advertising contract, the problem is solved in the same way. Figure 6 shows the results for different states of the problem. Table 3 summarizes the results of the second period.

*G (0.41)*

| | R | | TV | |
|---|---|---|---|---|
| R | 9859 | 11738 | 6518 | 12349 |
| TV | 10371 | 7767 | **6961** | **8293** |

*G (0.43)*

| | R | | TV | |
|---|---|---|---|---|
| R | 9859 | 11738 | 6829 | 11956 |
| TV | 10050 | 8144 | **6961** | **8293** |

*G (0.45)*

| | R | | TV | |
|---|---|---|---|---|
| R | **9859** | **11738** | 7138 | 11562 |
| TV | 9728 | 8521 | 6961 | 8293 |

**Fig. 6. The Equilibrium of G(0.41), G(0.43) and G(0.45) in the Presence of Contract**

**Table 3. The Result of Stage 2 (second period) in the Presence of Cooperative Advertising**

| State ($\eta$) | Equilibrium solution | Equilibrium payoff |
|---|---|---|
| 0.41 | (TV,TV) | (6961,8293) |
| 0.43 | (TV,TV) | (6961,8293) |
| 0.45 | (R,R) | (9859,11738) |

The payoff table of the first period is presented in Figure 7-(a), and, equivalently, in Figure 7-(b). Similarly, in this case, the Nash equilibrium is (TV, R). Choosing (TV, R) in the first period will result in playing *G(0.45)* in the second period, and the equilibrium will be (R, R) according to Table 3. Therefore, the best strategy for retailers in the presence of cooperative advertising is playing (TV, R) and (R, R) in the first and second periods, respectively.

a)

| | R | | TV | |
|---|---|---|---|---|
| R | **6961** + 9859 | **8293** + 11738 | **6961** + 7159 | **8293** + 12218 |
| TV | **9859** + 10262 | **11738** + 8528 | **6961** + 7519 | **8293** + 8955 |

↓

b)

| | R | | TV | |
|---|---|---|---|---|
| R | 1682<br>0 | 2003<br>1 | 1412<br>0 | 2051<br>1 |
| TV | **2012<br>1** | **2026<br>6** | 1448<br>0 | 1724<br>8 |

**Fig. 7. (a) Payoff Table of Stage 1 in the Presence of Contract; (b) the Equivalent Payoff Table**

The decision variables, including the retailers' advertising levels and the manufacturer's participation rates are obtained as following: $a^1$=179.2, $a^2$=478.5, $t_1^{\,1}$=0.681 and $t_1^{\,2}$=0.714 for the retailer 1; $b^1$=482.9, $b^2$=506.25, $t_2^{\,1}$=0.706 and $t_2^{\,2}$=0.666 for the retailer 2.

**Comparative Analysis**

The manufacturer's participation in paying a percentage of retailers' advertising investments increases the retailers' incentive to spend more on advertising. Consider the equilibrium decision variables in the two discussed setting. For instance, the corresponding TV advertising investment of retailer 1 at the first period in the absence of contract is 40×18.27=731, which is increased to 40×179.2=7169 in the presence of contract. Furthermore, his investment at the second period is 781 and is increased to 9570 by offering the contract. Similarly, the advertising investments of retailer 2 at two periods are 832 and 1125, which is increased to 9658 and 10125 in the presence of the cooperative advertising program.

This increase in advertising investment leads to an increase in total demand from 879 and 988 in the 2 periods to 2202 and 2519; this also increases the profits of the members. The summary of the results of the absence and presence of cooperative advertising contract is shown in Table 4 and Table 5. Offering the contract improves the manufacturer's profit by 74% and leads to 138% improvement on each retailer's profit.

**Table 4. Comparison of Equilibrium Pair of Advertising Option in Two Cases**

| Period | Absence of contract | Presence of contract |
|---|---|---|
| 1 | (TV,R) | (TV,R) |
| 2 | (R,R) | (R,R) |

**Table 5. Comparison of Payoffs in Two Cases**

| Player | Absence of contract | Presence of contract | % Improvement |
|---|---|---|---|
| Manufacturer | 23498 | 41022 | 74 |
| Retailer 1 | 8435 | 20121 | 138 |
| Retailer 2 | 8504 | 20266 | 138 |

**Sensitivity Analysis of the Win-Win Condition**

It was shown that offering the contract improves the players' profit. For further analysis, the sensitivity of the win-win condition to the changes of $\eta$, $k_2$, and $c_2$ is investigated in the following. All parameters are constant during the first period, and some parameters are changed in the second period. Also, It is assumed that we have $\eta' = \eta - 0.02$ and $\eta'' = \eta + 0.02$ in the second period. Figure 8 shows the area outside the win-win condition, that is where at least one player is worse-off by offering contract. It is reasonable for the manufacturer to offer the contract only in the win-win condition. The results obtained according to changes in parameters are depicted in Figure 8. See Figure 8-(a) based on changing $\eta$ and $k_2$ for $c_2$=120; Figure 8-(b) based on changing $\eta$ and $c_2$ for $k_2$=50;

Figure 8-(c) based on changing $c_2$ and $k_2$ for $\eta=0.45$; and Figure 8-(d) based on changing $\eta$, $c_2$ and $k_2$. The determined space between two surfaces is outside the win-win condition.

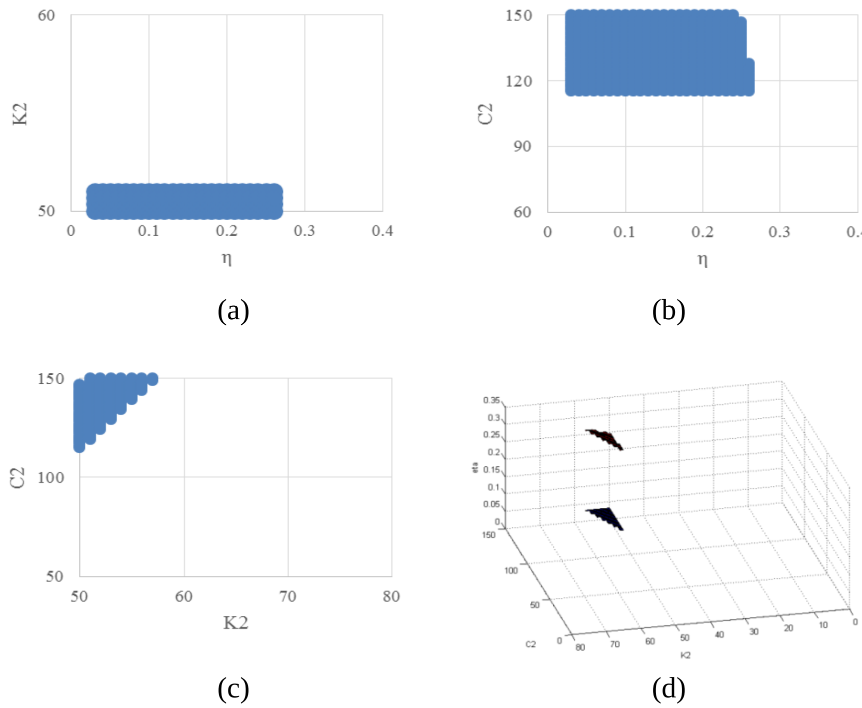


**Fig. 8. The Area (Space) Outside the Win-Win Condition According to Changes on: a) η and $k_2$; b) η and $c_2$; c) $c_2$ and $k_2$; d) η, $c_2$ and $k_2$.**

## Conclusions

Cooperative advertising has been highly promoted in companies such as IBM and Apple. Furthermore, a "two-period" phenomenon often occurs in the consumer-electronics industry when a new style of a product is launched. In this paper, the issue of cooperative advertising with advertising options is investigated in a two-period horizon in which retailers' market share in the 2nd period depends on the decisions made in the 1st period. The two-period cooperative advertising problem in this paper can be applied for mobile phones, personal computers, etc. in a retailer duopoly.

The main contributions of this paper are as following: (a) the cooperative advertising problem with two local advertising options is considered; (b) the two-period problem is investigated using stochastic games and dynamic programming; (c) two cases of absence and presence of cooperative advertising are compared in order to identify win-win conditions. First, the single-period problem is formulated, and solution is presented for the absence and presence of cooperative advertising contract. Then, the single-period problem is extended to two-period case, and an algorithm is proposed to solve the problem.

A two-period example is provided and solved for two cases of the absence and presence of cooperative advertising contract. Our comparative analysis shows that retailers' investments are increased by offering the contract. That leads to about 150% increase in the total market demand of each period. Consequently, offering the contract improves the manufacturer’s profit by 74% and leads to 138% improvement in each retailer's profit. Using sensitivity analysis, we found that the cooperative advertising contract is not always Pareto improving for the channel. However, if the contract is offered in the win-win condition, the players’ profit will be increased significantly. For further research, one can propose a hybrid mechanism that can be a win-win strategy for all players in all parameter settings.

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