An approach based slack variables in network data envelopment analysis to incorporate dynamic effects

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ARTICLE INFO	ABSTRACT
Article type: Research Article **Article History:** Received 16 March 2023 Revised 02 July 2023 Accepted 16 July 2023 Published Online 12 June 2024 **Keywords:** *Efficiency,* *Network Data Envelopment Analysis,* *Dynamic effects,* *Slacks-based model.*	Senior managers are the customers of organizational performance measurement methods to help them make better decisions at the firm level. One of the most applicable methods is Network Data Envelopment Analysis (DEA). Network DEA models consider systems that have a network structure in which system inputs, after passing several intermediate interactions, are transformed into intermediate productions and finally leave the system as output products. However, many real-world cases do not necessarily conform to this network structure, which is related to the system outputs during multiple time periods or the same "dynamic" impacts. These structures cannot handle dynamic impacts. Therefore, this paper presents a novel structure that can consider the dynamic impacts and influences of sub-units on each other at various time periods. Besides, two models based on slack variables are proposed which can consider dynamic effects and calculate the efficiency of such networks. Using these models, the overall efficiency of networks is calculated for the whole time period. Finally, these models are applied to two examples, and the results obtained are compared with other methods.

1. Introduction

The organizational efficiency measurement is an important task in management. Senior managers tend to show the past accomplishments of decision making units (DMUs) to determine different efficiency gain sources for their firms and reveal suitable directions for future development.

Data envelopment analysis (DEA), introduced by Farrel [1] and then extended by Chames et al. [2], is a nonparametric method to measure the efficiency of a DMU as a whole unit without considering its internal structure. In traditional DEA, each organization is taken into account as a black box consisting of the inputs consumed and the outputs produced by the organization and overlook the operations of the internal components to measure efficiency. Because of the interrelationships of the components, the inefficiency of any component may have negative impacts on the system overall performance, leading to the deviation of the efficiency measures from reality.

In order to tackle this issue, network DEA was introduced by Färe and Grosskopf [3]. The basic idea of the network models is that the inefficiencies of an organization's subunits affect the overall efficiency. Network DEA is distinguished by several characteristics, including breaking down the efficiency of a DMU into the efficiency of each one of its subunits and calculating the efficiency of large and complex organizations, considering the organization as a network of series, parallel or mixed structures, and finally calculating networks with circuits. Lothgren and Tambour [4] applied a network DEA model to measure the efficiency and productivity of Swedish pharmacies that consider customer satisfaction. They compared the results of the network DEA model with traditional DEA models. Lewis and Sexton [5] developed a network DEA model that had a multi-stage structure. Liang et al. [6] presented a network DEA model to measure the efficiency of a supplier and a retailer in a two-echelon supply chain in coordination or a leader-follower relationship. Prieto and Zofio [7] optimized primary input allocations, intermediate products, and final demand products by network DEA techniques and succeeded in applying their models to the input-output database of OECD countries. Yu and Lin [8] used a network DEA model to evaluate the performance of European railways. The proposed model allocates common inputs (e.g., the number of employees and length of the line) to different railway processes, such as passenger, freight, and consumption processes. Kao [9] developed a relational network model and considered the interrelations between various sub-processes: series, parallel, and mixed models. Tone and Tsutsui [10] formulated network structures using slacks that evaluated the overall efficiency of DMUs along with divisional efficiencies. Fukuyama and Weber [11] applied a network directional distance function to enable network performance to be measured when firms produce bad outputs. Hatami-Marbini and Saati [12] proposed a new common-weights DEA model for the network structure by considering the interrelationship of the processes within the production system to evaluate the technical efficiency of the whole system and their processes. Shadab et al. [13] introduced sustainable supply chains of resin manufacturing companies in Iran as a two-stage supply chain and identified four scenarios in which congestion can occur in intermediate products. For each scenario, they presented a dominant cone definition in the network structure and found out the status of congestion in terms of the power of intermediate products by their proposed network DEA. Michali et al. [14] extended the network DEA additive decomposition approach to investigate intermediate and undesirable outputs.

Färe and Grosskopf [15] introduced a dynamic structure as a special type of series structure that consisted of various single-period technologies connected by storable inputs and carry-over outputs from one period to another. Jaenicke [16] proposed a dynamic DEA model to investigate the role of soil capital in observed productivity growth and the rotation effect at a farm in Pennsylvania. Emrouznejad and Thanassoulis [17] developed a DEA model to assess 15 universities in the UK from 1994 to 1998. This model measures the comparative performance of DMUs when their input-output levels are described by inter-temporal dependencies. Tone and Tsutsui [18] proposed a dynamic slacks-based performance measure by categorizing four kinds of carry-over activities, including good, bad, free, and fixed. Wang et al. [19] utilized a DEA model to measure the total-factor energy and environmental efficiency of 29 administrative regions and three areas of China during the period of 2000–2008. Wang et al. [20] identified the key factors that affected energy efficiency from the perspectives of management level, productive scale, and technical progress to utilize DEA models from both static and dynamic perspectives. Khalili-Damghani et al. [21] presented a dynamic multi-

stage DEA approach to assess energy consumption in cotton production. This model determines the overall efficiency of DMUs with dynamic inputs.

Chen [22] presented a network DEA model with a novel efficiency measure considering dynamic effects in production networks. Bogetoft et al. [23] proposed a dynamic network DEA model to examine the optimal investment path for U.S. manufacturers while maximizing the final outputs throughout the investigation periods. Chen [24] developed a dynamic multi-activity network DEA model to determine the performance of farrow-to-finish swine production in Taiwan. Tone and Tsutsui [17] developed a network DEA model combined with dynamic effects. This model enables the overall efficiency of DMUs over the entire observed period and evaluates dynamic changes in the period efficiency and divisional efficiency of DMUs. Fukuyama and Weber [25] proposed a dynamic two-stage network model for Japanese commercial banks. Omrani and Soltanzadeh [26] developed a relational dynamic network DEA model that simultaneously measures the efficiencies of the system and its internal processes over time. They used the proposed model to calculate the efficiency scores of eight Iranian airlines over several periods. Zhang et al. [27] used a Russell multi-activity network DEA model to analyze the problems of intermediates, shared input, and slack-based measures in a unified framework. They provided policymakers with process-specific information when investigating the innovation performance of China's high-tech industries. Shao et al. [28] developed a two-stage network DEA model that considers production efficiency, wastewater treatment efficiency, and waste gas treatment efficiency to analyze the eco-efficiencies of different industrial sectors. Salehzadeh et al. [29] applied DEA by considering dynamic effects to measure organizational efficiency at various periods based on a balanced scorecard model. Alizadeh et al. [30] proposed a dynamic network-based DEA approach to measure the performance of each sub-system process and the entire system in multi-period planning horizons. Lahouel et al. [31] used a slacks-based dynamic network DEA approach to assess the financial stability of banks, including non-performing loans, as a measure of risk that is carried over between two consecutive periods. Huang et al. [32] developed a modified slacks-based dynamic network DEA model to measure the overall production and service efficiencies of forest parks. Fukuyama et al. [33] used dynamic network DEA by proposing a sequential structure incorporating dual-role characteristics of the production factors and also behavioral-causal analysis. They applied this proposed approach to the banking industry.

This paper presents two models, including Simple Slack Base Measure (SSBM) and Allocation Slack Base Measure (ASBM), in three forms, input-oriented, output-oriented, and non-oriented, to formulate the network structure. The main contributions of this paper are to develop models to measure DMUs's efficiency with network structures in which there is cycles and allocated links in a dynamic setting over several time periods. Cycle links refer to the links that constitute a cycle in a network, and allocated links are the links or intermediate products which are allocated to several sub-units. The developed models can determine optimal values for the inputs and outputs in addition to measuring the efficiency over several time periods. Besides, these models take into account the dynamic effect of the reduction of outputs due to the coefficient of perishability and indirectly reduce the organization's efficiency by considering the virtual units. These units in the continuous network structure provide the possibility of applying the dynamic effect in periods other than the final period, as well as for each SDMU.

The rest of the paper is organized as follows. Section 2 presents different possible links and connections with their constraints. Section 3 extends the SSBM and ASBM models. Section 4 presents a novel efficiency measuring approach. Two numerical results are solved to verify the effectiveness of the proposed models in Section 5. Finally, concluding remarks and directions for future research are discussed in Section 6.

2. Description of network structure with dynamic effects

This section gives a brief introduction to the conventional DEA, network DEA, and dynamic network DEA. Conventional DEA models consider DMUs as black box that has inputs and outputs (See Fig. 1a). These models are unable to incorporate internal processes, while the inefficiency of each DMU is often a result of sub-DMUs' (SDMU) inefficiencies. The network DEA was introduced to analyze and evaluate similar DMUs' inefficiencies considering their internal processes (Fig. 1b). Models introduced in network DEA incorporate the relationship between SDMUs as well as early inputs and

final outputs. After introducing network DEA, the dynamic effect is introduced. Fig. 1c represents a network structure with dynamic effects [34].
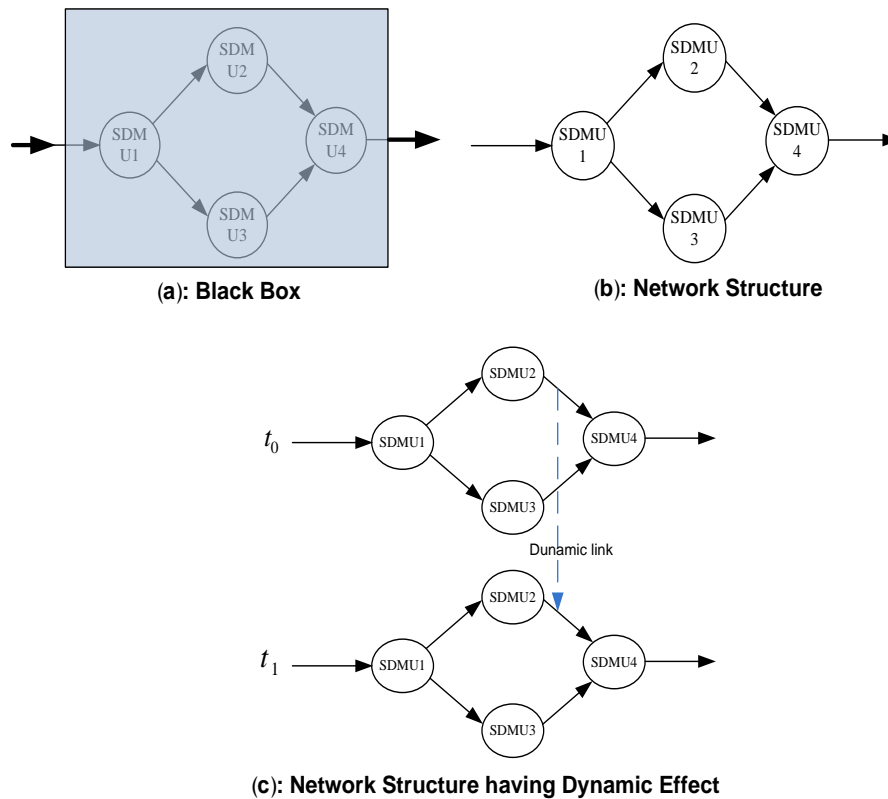

(**a**): **Black Box**

(**b**): **Network Structure**

(**c**): **Network Structure having Dynamic Effect**

Fig. 1. Different structures used in DEA.

In a dynamic structure, a fraction of the produced intermediate output is stored by the sub-divisions to be consumed during the next period. The stored intermediate outputs can be consumed in one or several periods. For instance, inter-organizational training courses are differently influential on different people and occur at several time periods. There may be several dynamic links in a network. Therefore, methods used for effectiveness assessment must be flexible enough to handle such links.

2.1. Assumptions and notations
In this subsection, assumptions and notations are introduced to formulate network structures with dynamic effects in three states consisting of input-oriented, output-oriented, and non-oriented as mathematical programming problems.

It is assumed that a DMU with a similar structure has K SDMUs in T time periods. The output of each sub-division can exit the system as the final output or can be used by other sub-divisions as an intermediate product or both. Similarly, inputs to each sub-division can be an initial input from outside of DMUs or can be intermediate outputs of other sub-divisions or a combination of these two states. Firstly, all the notations used throughout the modeling of the paper will be presented as follows.

Indexes
j, s The index of DMU ($j, s = 1,2, \dots, n$)
k The index of SDMU ($k = 1,2, \dots, K$)
t, t', t'' The index of period ($t = 1,2, \dots, T$)
i The index of input of sub-division k^{th} ($i = 1,2, \dots, I_k$)
l The index of output of sub-division k^{th} ($l = 1,2, \dots, O_k$)
m The index of intermediate input of sub-division k^{th} ($m = 1,2, \dots, I_k^Z$)
r The index of intermediate output of sub-division k^{th} ($r = 1,2, \dots, O_k^Z$)

Parameters

x_{ik}^{jt}	The ith initial input to SDMU k of DMU j in period t
λ_{kj}^{t}	The proportion the SDMU k of DMU j in period t to construct the reference division
z_{rkp}^{jt}	The rth intermediate output from SDMU k to SDMU p of DMU j in period t
$z_{mpk}^{jt}1$	The mth intermediate input to SDMU k from SDMU p of DMU j in period t
y_{lk}^{jt}	The lth final output of SDMU k of the DMU j in period t
$x_i^{jt}1$	The ith allocated input of DMU j in period t
$x_{ik}^{jt°}1$	The ith allocated input to SDMU k of DMU j in period t
$z_{rk}^{jt}1$	The rth allocated output of SDMU k of DMU j in period t
$z_{rkp}^{jt°}$	The rth allocated output of SDMU k to SDMU p of DMU j in period t
α	The percentage of the consumed intermediate output during $(0 \leq \alpha \leq 1)$ in period. $(1 - \alpha)$ is the remaining of the output transferred to be used during the next periods
y_l^{jt}	The lth allocated output of the DMU j in period t
$y_{lk}^{jt°}$	The lth allocated output of SDMU k of DMU j in period t
β	Drop coefficient (The effectiveness of the unconsumed intermediate output)

Set of index

D_k	$\{(p, v_{pk}, v_{kp}) \mid p \in K, \ v_{pk}, v_{kp} \in \mathbb{R}^+\}$ represents k^{th} sub-division where p is the index of sub-division, v_{pk} is the value of intermediate products from sub-division p to k, and v_{kp} is the value of products made by sub-division k consumed by p.
R_1	$= \{(k,i) \mid \ni \bar{x}_{ik}^{jt°} \ OR \ x_{ik}^{jt°} \subseteq x_i^{jt}\}$
R_2	$= \{q \mid \ni \bar{z}_{rkq}^{jt°} \ OR \ z_{rkq}^{jt°} \subseteq z_{rk}^{jt}\}$
R_3	$= \{q \mid \ni \bar{z}_{rkq}^{jt°} \ OR \ z_{rkq}^{jt°} \subseteq z_{rk}^{jt}\} \cup \{k \mid \ni \bar{y}_{lk}^{jt°} \ OR \ y_{lk}^{jt°} \subseteq y_l^{jt}\}$

Variables

$\bar{x}_{ik}^{jt}$	The ith input variable to the SDMU k of DMU j in period t
$\bar{x}_{ik}^{jt°}$	The ith input allocated variable to SDMU k of DMU j in period t
$\bar{z}_{mpk}^{jt}$	The mth intermediate product variable to SDMU k from SDMU p of DMU j in period t
$\bar{z}_{rkp}^{jt°}$	The rth intermediate output allocated variable from SDMU k to SDMU p of DMU j in period t
$\bar{y}_{lk}^{jt}1$	The lth output variable of SDMU k of DMU j in period t
$\bar{y}_{lk}^{jt°}1$	The lth output allocated variable of SDMU k of DMU j in period t
$S_{ik}^{t-}1$	The ith slack variable of SDMU k in period t in input-oriented model
S_{lk}^{t+}	The lth slack variable of SDMU k in period t in output-oriented model
θ_I	The resulting input-oriented efficiency of DMU j at simple dynamic model
$\dfrac{1}{\theta_O}$	The resulting output-oriented efficiency of DMU j at simple dynamic model
θ_N	The resulting non-oriented efficiency of DMU j at simple dynamic model
θ_I^T	The resulting input-oriented efficiency of DMU j at allocated dynamic model
$\dfrac{1}{\theta_O^T}$	The resulting output-oriented efficiency of DMU j at allocated dynamic model
θ_N^T	The resulting non-oriented efficiency of DMU j at allocated dynamic model

    All variables must satisfy the non-negativity condition. It is assumed that sub-division k has I_k initial inputs, O_k final outputs, I_k^Z intermediate inputs, and O_k^Z intermediate outputs. It is noteworthy to mention that the whole models are the constant returns-to-scale. In addition to simplicity, it is considered $E^{jt} = \alpha z_{rk}^{jt}$, $F^j = \beta(1-\alpha)z_{rk}^{jt}$, $\bar{E}^{jt} = \alpha\bar{z}_{rk}^{jt}$ and $\bar{F}^j = \beta(1-\alpha)\bar{z}_{rk}^{jt}$.

2.2. The types of links and their constraints

How the sub-divisions of an organization connect has a great impact on its overall performance. The nature of such links and connections depends on existing processes. Realizing the structure of real-world networks seems to be inevitable to model them. The proposed models consider the existing and desired conditions as constraints. To satisfy the desired conditions, the model calculates the value of each link to get the best performance for each SDMU, as well as DMU's efficiency, and the share of each sub-unit in comparison with the reference unit. In other words, the optimal values of links and the efficiency will be simultaneously calculated. But, to meet the existing conditions, the optimal values of inputs, outputs, and intermediate products are calculated after measuring the optimal share of each SDMU in comparison with the reference unit.

In this paper, links are divided into four groups including ordinary, allocated, dynamic, and aggregate ones [31], which are discussed in more detail below.

2.2.1. Ordinary links

The ordinary links consist of initial and intermediate inputs, as well as intermediate and final outputs. These links have three forms: (1) their values are totally transferred, (2) they only enter into one SDMU (input state) or only exit from one SDMU (output state), and (3) they take place just at one time period. To incorporate these links into the model, some constraints should be defined as Eq. (1) to (3) for SDMU k of division j as follows:

$$x_{ik}^{jt} \geq \sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t}, \qquad i=1,\ldots,I_k, \ t=1,\ldots,T \tag{1}$$

$$\overline{x}_{ik}^{jt} \geq \sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t}, \quad i=1,\ldots,I_k, \ t=1,\ldots,T \tag{2}$$

$$\overline{x}_{ik}^{jt} \leq x_{ik}^{jt}, \qquad\qquad i=1,\ldots,I_k, \ t=1,\ldots,T \tag{3}$$

The existing and desired constraints are introduced as Eq. (4) to (6).

$$\overline{z}_{mpk}^{jt} \geq \sum_{s=1}^{n} z_{mpk}^{st} \lambda_{ks}^{t}, p \in D_k, \ m=1,\ldots I_k^Z, \ t=1,\ldots,T \tag{4}$$

$$\overline{z}_{rkp}^{jt} \leq \sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ps}^{t}, p \in D_k, \qquad r=1,\ldots, O_k^Z, \ t=1,\ldots,T \tag{5}$$

$$\sum_{s=1}^{n} z_{mpk}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ps}^{t}, \qquad m=1,\ldots I_k^Z, \ r=1,\ldots, O_k^Z, \ t=1,\ldots,T \tag{6}$$

Symmetric equations are the same in both existing and desired states.

$$y_{lk}^{jt} \leq \sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t}, \quad t=1,\ldots,T, \ l=1,\ldots,O_k \tag{7}$$

$$\overline{y}_{lk}^{jt} \leq \sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t}, \quad t=1,\ldots,T, \ l=1,\ldots,O_k \tag{8}$$

$$\overline{y}_{lk}^{jt} \geq y_{lk}^{jt}, \qquad\qquad t=1,\ldots,T, \ l=1,\ldots,O_k \tag{9}$$

2.2.2. Allocated links

Inputs or outputs, which are allocated to several SDMUs in a time period, are called the allocated links. An allocated link is included in three forms of initial and intermediate inputs or intermediate and final outputs as depicted in Fig. 2.
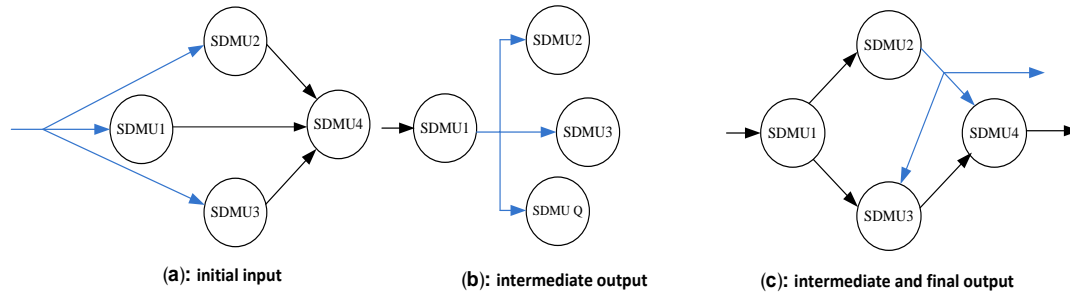
Fig 2. Different kinds of allocated links.

a. Initial inputs
Fig. 2a shows an initial input which is allocated to several SDMUs. Its corresponding constraints are represented by Eq. (10) to (12):

$$x_{ik}^{jt\circ} \geq \sum_{s=1}^{n} x_{ik}^{st\circ} \lambda_{ks}^{t}, \qquad (k,i) \in R_1, \qquad t = 1,\ldots,T \tag{10}$$

$$\overline{x}_{ik}^{jt\circ} \geq \sum_{s=1}^{n} x_{ik}^{st\circ} \lambda_{ks}^{t}, \qquad (k,i) \in R_1, \qquad t = 1,\ldots,T \tag{11}$$

$$x_{i}^{jt} \geq \sum_{k \in R_1} \overline{x}_{ik}^{jt\circ}, \qquad (k,i) \in R_1, \qquad t = 1,\ldots,T \tag{12}$$

b. Intermediate output
If the rth intermediate output of SDMU k is allocated to q SDMUs (See Fig. 2b) and $Z_{rkq}^{jt\circ}$ is the value allocated to SDMU q, $(q \in R_2)$ then we have:

$$z_{rkq}^{jt\circ} \geq \sum_{s=1}^{n} z_{rkq}^{st\circ} \lambda_{qs}^{t} \qquad t = 1,\ldots,T \tag{13}$$

$$\overline{z}_{rkq}^{jt\circ} \geq \sum_{s=1}^{n} z_{rkq}^{st\circ} \lambda_{qs}^{t}, \qquad t = 1,\ldots,T \tag{14}$$

$$\sum_{s=1}^{n} z_{rk}^{st\circ} \lambda_{ks}^{t} = \sum_{q \in R_2} \sum_{s=1}^{n} z_{rkq}^{st\circ} \lambda_{qs}^{t}, \qquad \forall t = 1,\ldots,T \tag{15}$$

$$\overline{z}_{rk}^{jt} = \sum_{q \in R_2} \overline{z}_{rkq}^{jt\circ}, \qquad \forall t = 1,\ldots,T \tag{16}$$

c. Intermediate and final output
If SDMU k has one final output and q intermediate outputs and $z_{rk}^{jt\circ}$ is the rth intermediate outputs to SDMU k of DMU j in period t, then $z_{rk}^{jt\circ}$ is divided into two values. $y_{lk}^{jt\circ} (\overline{y}_{lk}^{jt\circ})$ exits the DMU as the final output and its remaining is allocated to q SDMUs (Fig. 2c). Now, if $z_{rkq}^{jt\circ}$ is the value allocated to q $(q \in R_3)$, then we have:

$$z_{rkq}^{jt\circ} \geq \sum_{s=1}^{n} z_{rkq}^{st\circ} \lambda_{qs}^{t}, \qquad t = 1,\ldots,T \tag{17}$$

$$\overline{z}_{rkq}^{jt\circ} \geq \sum_{s=1}^{n} z_{ikq}^{st\circ} \lambda_{qs}^{t}, \qquad t = 1,\ldots,T \tag{18}$$

$$z_{rk}^{jt\circ} \leq \sum_{s=1}^{n} z_{rk}^{st\circ} \lambda_{ks}^{t}, \qquad t = 1,\ldots,T \tag{19}$$

$$\overline{z}_{rk}^{jt\circ} \leq \sum_{s=1}^{n} z_{rk}^{st\circ} \lambda_{ks}^{t}, \qquad t = 1,\ldots,T \tag{20}$$

$$y_{lk}^{jt\circ} \leq \sum_{s=1}^{n} y_{lk}^{st\circ} \lambda_{ks}^{t}, \qquad t = 1,\ldots,T \tag{21}$$

$$\overline{y}_{lk}^{jt\circ} \leq \sum_{s=1}^{n} y_{lk}^{st\circ} \lambda_{ks}^{t}, \qquad\qquad t = 1,\dots,T \tag{22}$$

$$\sum_{s=1}^{n} z_{rk}^{st\circ} \lambda_{ks}^{t} = \sum_{q \in R_3} \sum_{s=1}^{n} z_{rkq}^{st\circ} \lambda_{qs}^{t} + \sum_{s=1}^{n} y_{lk}^{st\circ} \lambda_{ks}^{t} \quad t = 1,\dots,T \tag{23}$$

$$\overline{z}_{rk}^{jt\circ} = \sum_{q \in R_3} \overline{z}_{rkq}^{jt\circ} + \overline{y}_{lk}^{jt\circ}, \qquad\qquad t = 1,\dots,T \tag{24}$$

2.2.3. Dynamic links

There are lots of real-world examples in which the outputs of an SDMU are consumed within several periods of time. For instance, corporations are forces to sell just some parts of their products based on selling polices and the remaining products are stored to be sold during the next time periods. These periods can be short or long. When the output of an SDMU is consumed during several periods, this state is called the dynamic state.

Suppose A and B are two SDMUs. Fig. 3 depicts these two SDMUs that belong to DMU j which is dynamic. Storing conditions of goods are not necessarily constant in real-world cases. Therefore, the inventory level is not constant during a period and usually decreases. Hence, a drop coefficient (β) is considered to make the model more realistic.
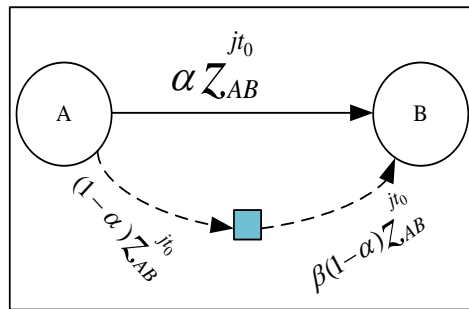

Fig. 3. A simple structure with dynamic link.

The dynamic link is discussed in two different states. The remaining outputs are consumed during the next period with the fixed quality ($\beta = 1$) or with the dropped quality ($0 \leq \beta < 1$). When $\beta = 1$, the dynamic link converts to the allocated link with a little difference. In the dynamic type, the intermediate output is allocated to several SDMUs during two or more time periods, but in the allocated state, the output is allocated to two or more SDMUs during the same time period. The drop coefficient β is the more important difference between the two links. In the allocated link, the summation of the allocated values equals the initial value, but this does not necessarily occur in the dynamic one. Therefore, the same constraints of the allocated link are utilized, but the variables are defined over various periods of time. However, to incorporate the dynamic effects into the model, a virtual DMU is defined (Fig. 4). A, B, and C represent a SDMU in this figure. This virtual DMU has several outputs at various periods.
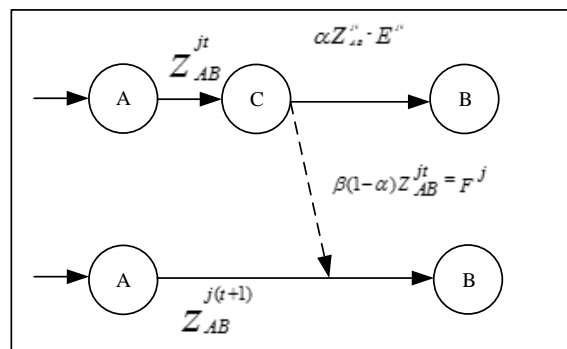

Fig. 4. Cumulative-allocated dynamic link.

Defining the virtual DMU has several benefits, including the fact that (a) dynamic impacts are transferred to the next periods and that (b) values of β influences the overall efficiency of the corporation during time periods. This can be proved very easily. A decrease in the output by the drop coefficient influences the efficiency of the virtual DMU, and since the virtual DMU is entered into the computations, its effect is induced to the corporation. Another benefit of defining the virtual DMUS is that whole links are incorporated into the calculations as valid.

After defining the virtual DMU, dynamic constraints of the second state will be as Eq. (25) to (34):

$$\bar{z}_{AB}^{jt} \leq \sum_{s=1}^{n} z_{AB}^{st} \lambda_{As}^{t} \tag{25}$$

$$\bar{z}_{AB}^{jt} \geq \sum_{s=1}^{n} z_{AB}^{st} \lambda_{Cs}^{t} \tag{26}$$

$$\bar{F}^{j} \leq \sum_{s=1}^{n} F^{s} \lambda_{Cs}^{t} \tag{27}$$

$$\bar{E}^{jt} \leq \sum_{s=1}^{n} E^{st} \lambda_{Cs}^{t} \tag{28}$$

$$\bar{E}^{jt} \geq \sum_{s=1}^{n} E^{st} \lambda_{Bs}^{t} \tag{29}$$

$$\sum_{s=1}^{n} z_{AB}^{st} \lambda_{As}^{t} = \sum_{s=1}^{n} z_{AB}^{st} \lambda_{Cs}^{t} \tag{30}$$

$$\bar{z}_{AB}^{j(t+1)} + \bar{F}^{j} \geq \sum_{s=1}^{n} (F^{s} + z_{AB}^{s(t+1)}) \lambda_{Bs}^{(t+1)} \tag{31}$$

$$\bar{z}_{AB}^{j(t+1)} \leq \sum_{s=1}^{n} z_{AB}^{s(t+1)} \lambda_{As}^{(t+1)} \tag{32}$$

$$\sum_{s=1}^{n} E^{st} \lambda_{Cs}^{t} = \sum_{s=1}^{n} E^{st} \lambda_{Bs}^{t} \tag{33}$$

$$\sum_{s=1}^{n} z_{AB}^{s(t+1)} \lambda_{As}^{(t+1)} + \sum_{s=1}^{n} F^{s} \lambda_{Cs}^{t} = \sum_{s=1}^{n} (F^{s} + z_{AB}^{s(t+1)}) \lambda_{Bs}^{(t+1)} \tag{34}$$

These constraints can be easily extended to several time periods.

2.2.4. Aggregate link
Aggregate value of a link during several time periods is called the aggregate link. Aggregate links are used to measure efficiency of DMUs during different time periods when the initial and optimal values of inputs and outputs are determined.

It should be noted that there are two kinds of constraints: existing and desired. In existing constraints, the resulting inputs and outputs of SDMUs are used, so two sides of the constraint have the determined coefficients while the values of the links between SDMUs are assumed the unknown in the desired constraints such that their optimal value is obtained after solving the model.

3. Dynamic models with slack-based measure approach
The slacks-based measure (SBM) is a non-radial method that is suitable to measure efficiencies when inputs and outputs may change non-proportionally. This method can decompose the overall efficiency into divisional ones. SBM returns an efficiency measure between 0 and 1. It takes the value of 1 if and only if the DMU concerned is on the frontiers of the production possibility set with no input/ output slacks. In that respect, SBM differs from traditional radial measures of efficiency that do not take the existence of slacks into account.

In this section, SSBM and ASBM models are presented for measuring the efficiency of DMUs with dynamic effects. These models can calculate the efficiency of networks, which have dynamic effects, a cycle link, and allocated links. The SSBM model has fewer variables and constraints compared with

the ASBM model. By solving the ASBM model, not only is the efficiency of DMUs calculated for the whole period but also the values of each link are determined so that each SDMU has the best performance. Using both models, the optimal values of inputs or output of DMUs, when assuming that its SDMUs are efficient, can be calculated. If it is assumed that SDMU k of DMU j has the input and output values of $(x_{ik}^{jt}, z_{kp}^{jt}, y_{lp}^{jt})$, then the optimal solution in the output-oriented state will be calculated as $(\sum_{s=1}^{n} z_{kp}^{st} \lambda_{ps}^{t}{}^{*}, \sum_{s=1}^{n} y_{lp}^{st} \lambda_{ps}^{t}{}^{*})$ and the optimal input in the input-oriented state will be calculated as $(\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t}{}^{*}, \sum_{s=1}^{n} z_{kp}^{st} \lambda_{ks}^{t}{}^{*})$.

3.1. Simple dynamic model with slack-based measure Approach
According to the assumptions already described, the SSBM model is introduced in the input-oriented, output-oriented, and non-oriented states. It is remarkable that some points must be considered regarding the above model: (1) the slacks of the objective function are only for the constraints of the initial inputs and each of the SDMUs during the whole period, (2) dynamic and allocated constraints will be added to the model according to what presented in subsection 2.2, and (3) allocated constraints, if any, will be added to the model.

3.3.1. Input-oriented state (SSBM-I)
To calculate the input-oriented efficiency of DMU j, the following linear programming (LP) problem should be solved:

$$\theta_I = \text{Min} \left[1 - \frac{1}{T\sum_{k=1}^{K} I_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \frac{S_{ik}^{t-}}{x_{ik}^{jt}} \right] \tag{35}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} + S_{ik}^{t-} = x_{ik}^{jt}, \quad i = 1,\ldots,I_k, \quad k = 1,\ldots,K, \ t = 1,\ldots,T \tag{36}$$

$$\sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t} \geq y_{lk}^{jt}, \quad l = 1,\ldots,O_k, \quad k = 1,\ldots,K, \ t = 1,\ldots,T \tag{37}$$

$$\sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\ldots,I_k^Z, \quad r = 1,\ldots,O_k^Z, \ k,p = 1,\ldots,K, \ t = 1,\ldots,T \tag{38}$$

3.1.2. Output-oriented state (SSBM-O)
The LP problem of the output-oriented state for DMU j is as below:

$$\frac{1}{\theta_O} = \text{Max} \left[1 + \frac{1}{T\sum_{k=1}^{K} O_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{l=1}^{O_k} \frac{S_{lk}^{t+}}{y_{lk}^{jt}} \right] \tag{39}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} \leq x_{ik}^{jt}, \quad i = 1,\ldots,I_k, \quad k = 1,\ldots,K, \ t = 1,\ldots,T \tag{40}$$

$$\sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t} - S_{lk}^{t+} = y_{lk}^{jt}, \quad l = 1,\ldots,O_k, \quad k = 1,\ldots,K, \ t = 1,\ldots,T \tag{41}$$

$$\sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\ldots,I_k^Z, \quad r = 1,\ldots,O_k^Z, \ k,p = 1,\ldots,K, \ t = 1,\ldots,T \tag{42}$$

3.1.3. Non-oriented state (SSBM-N)
The non-oriented state is obtained from the input and output-oriented states as below:

$$\theta_N = \text{Min} \frac{1 - \dfrac{1}{T\sum_{k=1}^{K} I_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \dfrac{S_{ik}^{t-}}{x_{ik}^{jt}}}{1 + \dfrac{1}{T\sum_{k=1}^{K} O_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{l=1}^{O_k} \dfrac{S_{lk}^{t+}}{y_{lk}^{jt}}} \tag{43}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} + S_{ik}^{t-} = x_{ik}^{jt}, \quad i = 1,\dots, I_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{44}$$

$$\sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t} - S_{lk}^{t+} = y_{lk}^{jt}, \quad l = 1,\dots, O_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{45}$$

$$\sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots, I_k^Z, \quad r = 1,\dots, O_k^Z, \quad k,p = 1,\dots, K, \quad t = 1,\dots,T \tag{46}$$

3.2. Allocated dynamic model with slack-based measure approach

The slack-based model presented here is much superior to the simple dynamic model. In addition to being capable of measuring the efficiency of DMUs and calculating the fraction of each SDMU in constructing the reference DMU, this model is capable of determining the values of each link such that each SDMU has the best performance. As with the simple dynamic model, this model is also presented in the input-oriented, output-oriented, and non-oriented states. Depending on the network structure and dynamic links in it, the constraints from 25 to 34 are added to any of the models.

3.2.1. Input-oriented state (ASBM-I)

The allocated input-oriented model for DMU *j* is an LP problem as below:

$$\theta_I^T = \text{Min} \left[1 - \frac{1}{T\sum_{k=1}^{K} I_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \frac{S_{ik}^{t-}}{x_{ik}^{jt}} \right] \tag{47}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} \leq \overline{x}_{ik}^{jt}, \quad i = 1,\dots, I_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{48}$$

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} \leq \overline{x}_{ik}^{jt}, \quad i = 1,\dots, I_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{49}$$

$$\overline{x}_{ik}^{jt} + S_{ik}^{t-} = x_{ik}^{jt}, \quad i = 1,\dots, I_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{50}$$

$$\overline{y}_{lk}^{jt} \geq y_{lk}^{jt}, \quad l = 1,\dots, O_k, \quad k = 1,\dots, K, \ t = 1,\dots,T \tag{51}$$

$$\overline{z}_{rkp}^{jt} \leq \sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t}, \quad r = 1,\dots, O_k^Z, \quad k,p = 1,\dots, K, \quad t = 1,\dots,T \tag{52}$$

$$\overline{z}_{mkp}^{jt} \geq \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots, I_k^Z, \quad k,p = 1,\dots, K, \quad t = 1,\dots,T \tag{53}$$

$$\sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots, I_k^Z, \quad r = 1,\dots, O_k^Z, \quad k,p = 1,\dots, K, \quad t = 1,\dots,T \tag{54}$$

$$\sum_{t'}^{t-1} \sum_{m=1}^{I_k^Z} \sum_{p=1}^{K} \overline{z}_{mpk}^{jt'} + \overline{z}_{mpk}^{jt} \leq \sum_{s=1}^{n} \lambda_{ks}^{t} \left(\sum_{t'}^{t-1} \sum_{m=1}^{I_k^Z} \sum_{p=1}^{K} z_{mpk}^{st'} + z_{mpk}^{jt} \right), \ t = 2,\dots T, \ k = 1,\dots K \tag{55}$$

3.2.2. Output-oriented model (ASBM-O)

The allocated output-oriented model for DMU *j* is as follows:

$$\frac{1}{\theta_O^T} = \text{Max} \left[1 + \frac{1}{T \sum_{k=1}^{K} O_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{l=1}^{o_k} \frac{S_{lk}^{t+}}{y_{lk}^{jt}} \right] \tag{56}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} \le \bar{x}_{ik}^{jt}, \qquad i = 1,\dots,I_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{57}$$

$$\sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t} \ge \bar{y}_{lk}^{jt}, \qquad l = 1,\dots,O_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{58}$$

$$\bar{x}_{ik}^{jt} \le x_{ik}^{jt}, \qquad\qquad i = 1,\dots,I_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{59}$$

$$\bar{y}_{lk}^{jt} - S_{lk}^{t+} = y_{lk}^{jt}, \ l = 1,\dots,O_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{60}$$

$$\bar{z}_{rkp}^{jt} \le \sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t}, \quad r = 1,\dots,O_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{61}$$

$$\bar{z}_{mkp}^{jt} \ge \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots,I_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{62}$$

$$\bar{z}_{mkp}^{jt} \ge \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots,I_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{63}$$

$$\bar{z}_{mkp}^{jt} \ge \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \quad m = 1,\dots,I_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{64}$$

3.2.3. Non-oriented model (ASBM-N)

Using the two allocated input-oriented and allocated output-oriented models, we define non-oriented model for DMU *j* as an LP problem as follows:

$$\theta_N^T = \text{Min} \frac{\left[1 - \dfrac{1}{T \sum_{k=1}^{K} I_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \dfrac{S_{ik}^{t-}}{x_{ik}^{jt}} \right]}{\left[1 + \dfrac{1}{T \sum_{k=1}^{K} O_k} \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{l=1}^{o_k} \dfrac{S_{lk}^{t+}}{y_{lk}^{jt}} \right]} \tag{65}$$

S.T.

$$\sum_{s=1}^{n} x_{ik}^{st} \lambda_{ks}^{t} \le \bar{x}_{ik}^{jt}, \qquad i = 1,\dots,I_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{66}$$

$$\sum_{s=1}^{n} y_{lk}^{st} \lambda_{ks}^{t} \ge \bar{y}_{lk}^{jt}, \qquad l = 1,\dots,O_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{67}$$

$$\bar{x}_{ik}^{jt} + S_{ik}^{t-} = x_{ik}^{jt}, \qquad i = 1,\dots,I_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{68}$$

$$\bar{y}_{lk}^{jt} - S_{lk}^{t+} = y_{lk}^{jt}, \ l = 1,\dots,O_k, \quad k = 1,\dots,K, \ t = 1,\dots,T \tag{69}$$

$$\bar{z}_{rkp}^{jt} \le \sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t}, \qquad r = 1,\dots,O_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{70}$$

$$\bar{z}_{mkp}^{jt} \ge \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, \qquad m = 1,\dots,I_k^Z, \ \ k,p = 1,\dots,K, \quad t = 1,\dots,T \tag{71}$$

$$\sum_{s=1}^{n} z_{rkp}^{st} \lambda_{ks}^{t} = \sum_{s=1}^{n} z_{mkp}^{st} \lambda_{ps}^{t}, m = 1,\dots,I_k^Z, \quad r = 1,\dots,O_k^Z, \ \ k,p = 1,\dots,K, \ \ t = 1,\dots,T \tag{72}$$

$$\sum_{t'}^{t+1} \sum_{r=1}^{O_k^Z} \sum_{p=1}^{K} \bar{z}_{rkp}^{jt'} + \bar{y}_{lk}^{jt} \le \sum_{s=1}^{n} \lambda_{ks}^{t} \left(\sum_{t'}^{t+1} \sum_{r=1}^{O_k^Z} \sum_{p=1}^{K} z_{rkp}^{st'} + y_{lk}^{jt} \right), t = 1,\dots T-1, \ k = 1,\dots,K \tag{73}$$

$$\sum_{t''}^{t-1}\sum_{m=1}^{I_k^z}\sum_{p=1}^{K}\bar{z}_{mpk}^{jt''} + \bar{z}_{mpk}^{jt} \le \sum_{s=1}^{n}\lambda_{ks}^{t}\left(\sum_{t''}^{t-1}\sum_{m=1}^{I_k^z}\sum_{p=1}^{K}z_{mpk}^{st''} + z_{mpk}^{jt}\right), \quad t = 2,\dots T, \quad k = 1,\dots K \tag{74}$$

SSBM and ASBM are always feasible. In following, feasibility of SSBM-O is proofed: There are two statuses. Status 1: If $x_{ik}^{jt} \ge$ average $\{x_{ik}^{st}|s = 1,..,n\}$ be confirmed for each i of sub-division such as k of DMU j^{th}, then assume that $\lambda_{ks}^{t} = \frac{1}{n}$. In this way, equation (40) becomes $\frac{1}{n}\sum_{s=1}^{n}x_{ik}^{st} \le x_{ik}^{jt}$. Based on equations (41) and (42), we can write $S_{lk}^{t+} = y_{lk}^{jt} + \frac{1}{n}\sum_{s=1}^{n}y_{lk}^{st}$ and $\frac{1}{n}\sum_{s=1}^{n}z_{rkp}^{st} = \frac{1}{n}\sum_{s=1}^{n}z_{mkp}^{st}$. The values of z_{rkp}^{st} and z_{mkp}^{st} are the same because the intermediate products enter on one side and leave on the other side. Thus, $S_{lk}^{t+} \ge 0$. Status 2: If there is i for a sub-division such as k of DMU j^{th} that $x_{ik}^{jt} <$ Average $\{x_{ik}^{st}|s = 1,..,n\}$ be confirmed, then assume that $\lambda_{kj}^{t} = 1$ and other λ_{ks}^{t} are zero. In this way, equations (40) to (42) are satisfied and thus $S_{lk}^{t+} \ge 0$. Hence, $(\lambda_{ks}^{t}, S_{lk}^{t+})$ is a feasible solution in two statuses.

4. The proposed approach to calculate efficiency

SSBM and ASBM are models based on slack variables and can consider dynamic, allocated, and cycle links and apply the dynamic effect in several time periods. These models cover the dynamic effect for measuring the efficiency of network structures over several time periods. The presence of virtual units in the continuous network structure provides the possibility of applying the dynamic effect in periods other than the final period as well as for each SDMU. The virtual units take into account the dynamic effect of the reduction of outputs due to the coefficient of perishability and indirectly reduce the organization. By using the proposed models, one can compare the organization's performance in different periods, identify inefficient resources and also make the necessary decisions to improve the organization's performance. These are the main merits and advantages of the proposed models.

These proposed models are based on network structures that will be the result of aggregation of networks in periods from period 0 to T through dynamic link as a unified network. The constraints of models are formed based on input, output, and intermediate products, and the dynamic effect will also play the same role as input for an SDMU. In a network structure, a DMU is overall efficient if and only if it is efficient for all SDMUs [17]. Besides, the efficiency of a DMU is obtained from the aggregation of the efficiency of SDMUs and the DEA model for an SDMU becomes a classical model, the feasibility of its solution is certain. Thus, the dual of each proposed model is transformed to a network model that has an optimal solution [3]. Since the dual has an optimal solution, the primal problem will also have an optimal solution.

To calculate the efficiency of any DMU with any kind of structure, some stages must be considered before selecting and using the models (classic or network). This section proposes an approach that can handle dynamic effects for various kinds of structures. The stages of this approach are shown in Fig. 5.

Step 1: Determine SDMUs and then the inputs and outputs of each SDMU.
Step 2: Draw the network including SDMUs and their inputs and outputs.
Step 3: Collect the required data of several DMUs or several periods related to a DMU.
Step 4: Determine the dynamic effect and the virtual unit.
Step 5: Draw Continuous network in periods along with dynamic effects.
Step 6: Select one of the following models to increase the efficiency.
      1) Output-oriented: Suitable for DMUs that seek to increase output without changing input.
      2) Input-oriented: Suitable for DMUs that seek to reduce input without changing output.
      3) Non-oriented: Suitable for DMUs that seek to decrease of input and increase of output, simultaneously.
Step 7: Determine the model:
      SSBM: Calculating efficiency.
      ASBM: Calculating efficiency and optimal values of links, simultaneously.
Step 8: Determine the constraints of the model: using constraints from 1 to 34 according to the type of links in the network.
Step 9: Solve the developed model.

Fig. 5. The proposed approach for obtaining the efficiency of network structures with dynamic effects.

It should be noted that the optimal input and output values for the whole SDMUs can be obtained by using λ^*. Here, it should be emphasized that the obtained efficiency of a DMU depends on the number of compared DMUs, so the use of classic DEA to obtain the efficiency of each SDMU can be helpful.

5. The results and discussion
This section presents two numerical examples with different specifications to measure efficiency by exploiting the proposed models.

5.1. Example 1 and its results
The first example consists of six DMUs with four SDMUs during two time periods $T = \{t_0, t_1\}$. Fig. 6 displays two continuous networks with allocated and ordinary links. Data related to DMUs of the small example are presented in Table 1. For instance, values of $x_{11}^{1t_0}$ and $z_{341}^{1t_1}$ are 2 and 4, respectively.

Table 1. Data related to DMUs of the small example.

| DMU	Input and output values in t_0						Input and output values in t_1						α	β
	x_1	y_1	y_2	y_3	y_4	y_5	x_1	y_1	y_2	y_3	y_4	y_5		
1	$x_{11}^{1t_0}$	$z_{121}^{1t_0}$	$z_{131}^{1t_0}$	$z_{241}^{1t_0}$	$z_{341}^{1t_0}$	$y_{14}^{1t_0}$	$x_{11}^{1t_1}$	$z_{121}^{1t_1}$	$z_{131}^{1t_1}$	$z_{241}^{1t_1}$	$z_{341}^{1t_1}$	$y_{14}^{1t_1}$	0.5	0.5
	2	1	1	2	5	3	1	1	2	4	4	2		
2	$x_{11}^{2t_0}$	$z_{121}^{2t_0}$	$z_{131}^{2t_0}$	$z_{241}^{2t_0}$	$z_{341}^{2t_0}$	$y_{14}^{2t_0}$	$x_{11}^{2t_1}$	$z_{121}^{2t_1}$	$z_{131}^{2t_1}$	$z_{241}^{2t_1}$	$z_{341}^{2t_1}$	$y_{14}^{2t_1}$	0.25	1
	1	3	1	10	12	10	2	4	2	7	6	10		
3	$x_{11}^{3t_0}$	$z_{121}^{3t_0}$	$z_{131}^{3t_0}$	$z_{241}^{3t_0}$	$z_{341}^{3t_0}$	$y_{14}^{3t_0}$	$x_{11}^{3t_1}$	$z_{121}^{3t_1}$	$z_{131}^{3t_1}$	$z_{241}^{3t_1}$	$z_{341}^{3t_1}$	$y_{14}^{3t_1}$	0.25	1
	4	5	2	6	3	8	1	2	4	5	5	6		
4	$x_{11}^{4t_0}$	$z_{121}^{4t_0}$	$z_{131}^{4t_0}$	$z_{241}^{4t_0}$	$z_{341}^{4t_0}$	$y_{14}^{4t_0}$	$x_{11}^{4t_1}$	$z_{121}^{4t_1}$	$z_{131}^{4t_1}$	$z_{241}^{4t_1}$	$z_{341}^{4t_1}$	$y_{14}^{4t_1}$	0.25	1
	1	2	2	3	8	9	2	1	1	4	3	10		
5	$x_{11}^{5t_0}$	$z_{121}^{5t_0}$	$z_{131}^{5t_0}$	$z_{241}^{5t_0}$	$z_{341}^{5t_0}$	$y_{14}^{5t_0}$	$x_{11}^{5t_1}$	$z_{121}^{5t_1}$	$z_{131}^{5t_1}$	$z_{241}^{5t_1}$	$z_{341}^{5t_1}$	$y_{14}^{5t_1}$	0.25	1
	3	3	4	7	7	4	1	5	4	3	7	6		
6	$x_{11}^{6t_0}$	$z_{121}^{6t_0}$	$z_{131}^{6t_0}$	$z_{241}^{6t_0}$	$z_{341}^{6t_0}$	$y_{14}^{6t_0}$	$x_{11}^{6t_1}$	$z_{121}^{6t_1}$	$z_{131}^{6t_1}$	$z_{241}^{6t_1}$	$z_{341}^{6t_1}$	$y_{14}^{6t_1}$	0.25	1
	2	2	3	8	4	7	3	2	4	5	4	2		
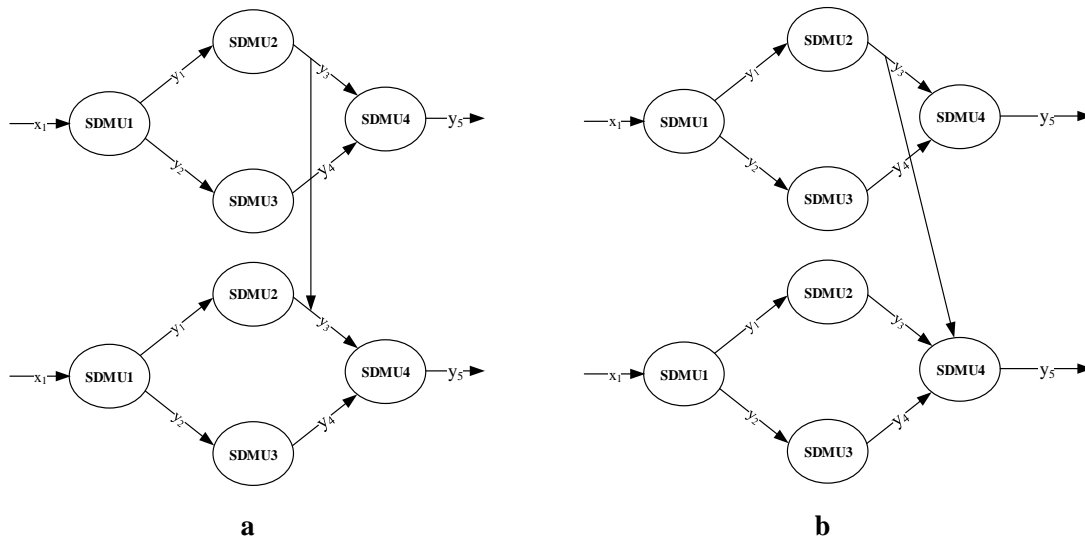
Fig. 6. A continuous network in two time periods with a) aggregate links and b) ordinary links

The proposed models in the output-oriented form are used to measure the efficiency of the continuous network with allocated links (Fig. 6a), as well as the simple ones (Fig. 6b). For example, the LP problem of SSBM-O for DMU 1 is as below:

$$\frac{1}{\theta_O} = \text{Max}\left[1 + \frac{1}{2(1)}\sum_{t=0}^{1}\frac{S_{14}^t}{y_{14}^{lt}}\right]$$

S.T.

$$\sum_{s=1}^{6}x_{11}^{st}\lambda_{1s}^t \le x_{11}^{lt},$$

$$\sum_{s=1}^{6}y_{14}^{st}\lambda_{4s}^t - S_{14}^{t+} = y_{14}^{lt},$$

$$\sum_{s=1}^{6}z_{121}^{st}\lambda_{1s}^t = \sum_{s=1}^{6}z_{mkp}^{st}\lambda_{2s}^t,$$

$$\sum_{s=1}^{6}z_{131}^{st}\lambda_{1s}^t = \sum_{s=1}^{6}z_{131}^{st}\lambda_{3s}^t,$$

$$\sum_{s=1}^{6}z_{131}^{st}\lambda_{1s}^t = \sum_{s=1}^{6}z_{131}^{st}\lambda_{3s}^t,$$

$$\sum_{s=1}^{6}z_{241}^{s0}\lambda_{2s}^0 = \sum_{s=1}^{6}z_{241}^{s0}\lambda_{Ms}^0,$$

$$\sum_{s=1}^{6}z_{341}^{s1}\lambda_{2s}^1 = \sum_{s=1}^{6}z_{341}^{s1}\lambda_{4s}^1,$$

$$\sum_{s=1}^{6}\alpha_s z_{241}^{s0}\lambda_{Ms}^0 = \sum_{s=1}^{6}\alpha_s z_{241}^{s0}\lambda_{4s}^0,$$

$$\sum_{s=1}^{6}(1-\alpha_s)\beta_s z_{241}^{s0}\lambda_{2s}^1 = \sum_{s=1}^{6}(1-\alpha_s)\beta_s z_{241}^{s0}\lambda_{4s}^0,$$

$$\lambda_{ks}^t \ge 0 \quad \forall k = 1,\dots 4, \quad s = 1,\dots,6 \quad t = 0,1$$

$$S_{14}^t \ge 0 \quad \forall t = 0,1$$

By solving this model, the efficiency inverse for DMU#1 becomes 19.7171 and so on for other DMUs. In finally, the results are compared with the hybrid method of Lewis et al. and Prieto et al. and also the method of Prieto et al. in Table 2 and Table 3. Based on these results, ASBM, SSBM, and Prieto et al.'s method have approximately the same values of efficiency inverse, but the hybrid method leads us to a somewhat smaller value. The causes of this difference can be described as follows: ASBM, SSBM, and Prieto et al.'s method consider the whole network as a model and measured the efficiency, but the hybrid method calculates the efficiency step by step beginning from the first SDMU and uses the optimal values for the following steps. Therefore, possible errors in these calculations affect the overall efficiency. In addition, how the network's interrelations are incorporated is effective in driving the results.

In general, it should be said that the mentioned methods obtained the efficiency inverse of DMUs as different while the overall arrangement of the inverse of DMU's efficiency is somewhat the same. Based on the obtained results from network with the aggregate link, all methods introduced DMU #4 and DMU#2 as the most efficient units. The output of the methods is not the same for the most inefficient unit. However, DMU #1 and #6 were identified as the most inefficient units with a different order by methods. These conclusions are also true for the network with ordinary links.

Table 2. The inverse of efficiency for DMUs by different methods for the network with ordinary links

	Prieto et al. [16]	Hybrid method of Lewis et al and Prieto et al. [31]	*SSBM*	*ASBM*
DMU1	19.5609	20	19.7171	19.7037
DMU2	2.6415	4	3.0308	3.0308
DMU3	6.6667	3.33	11.6416	11.6416
DMU4	2.8406	1.511	3.1447	3.1447
DMU5	6.67	6.67	15.3339	15.3339
DMU6	9.5427	20	22.9458	22.9458

Table 3. The inverse of efficiency for DMUs by different methods for the network with allocated links

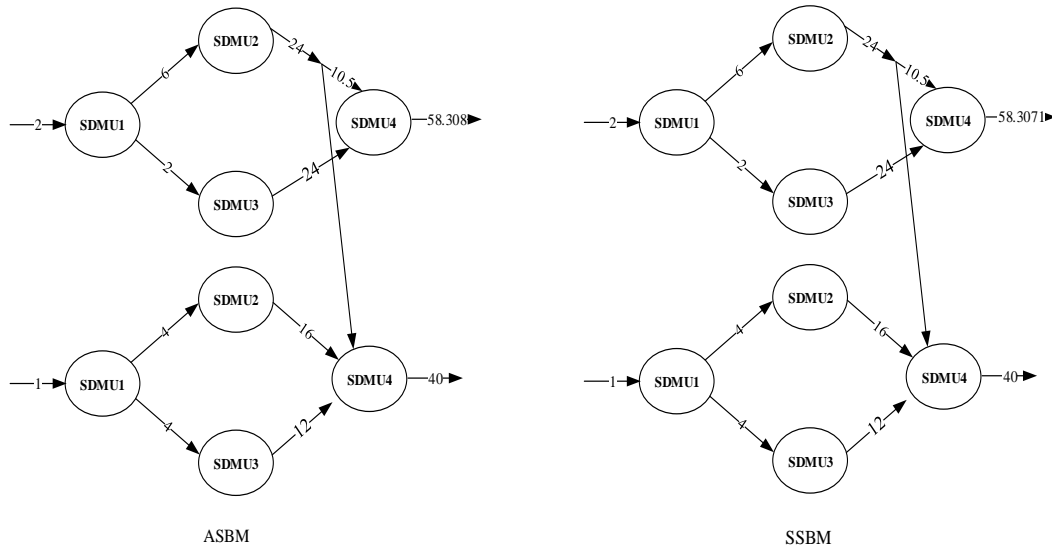|	Prieto et al. [16]	Hybrid method of Lewis et al and Prieto et al. [31]	*SSBM*	*ASBM*
DMU1	20	16	20.9391	20.9297
DMU2	3.3146	2.0615	5.0102	5.0420
DMU3	6.6667	8.9213	11.7492	11.7441
DMU4	3.6415	3.5556	5.1659	5.2276
DMU5	6.67	18	15.9493	15.9493
DMU6	9.6117	10.69	33.3517	33.3525

Fig. 7. The optimal values of the ordinary links calculated based on SSBM and ASBM
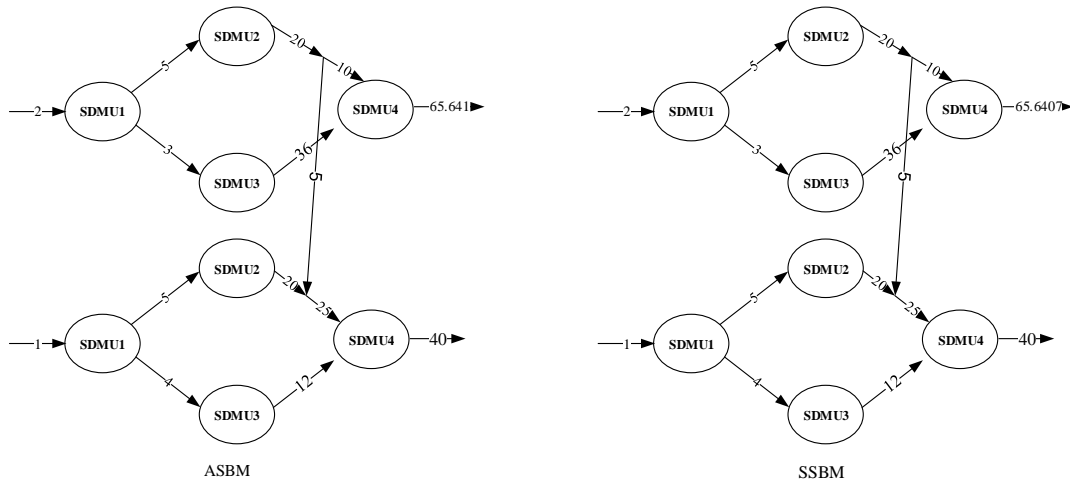

Fig. 8. The optimal values of the aggregate links calculated based on SSBM and ASBM.

Figs. 7 and 8 show the optimal values of links calculated based on SSBM and ASBM for the allocated links and the ordinary ones. These values are calculable for two other models. For example, $y_1^* = \sum_{s=1}^{6} z_{121}^{st_0} \lambda_{1s}^{*t_0} = 1 \times 0 + 3 \times 2 + 5 \times 0 + 2 \times 0 + 3 * 0 + 2 * 0 = 6$ is the optimal link value is SDMU #1 to SDMU #2. In fact, SDMU #1 must increase the intermediate product $z_{121}^{st_0}$ by y_1^* to reach the efficiency frontier. Besides, the optimal value of the final output will be $y_5^* = \sum_{s=1}^{6} y_{14}^{st_0} \lambda_{4s}^{*t_0} = 2 \times 0 + 10 \times 0 + 6 \times 0 + 10 \times 4 + 6 * 0 + 2 * 0 = 40$.

5.2. Example 2 and its results (Training effectiveness)

The example 2 is applied to measure the effectiveness of training courses. Improving the level of knowledge, skills, behavior, and insight of human resources is effective in the progress and development of organizational performance. To reach this aim, most organizations train their

employees. The effectiveness of training courses and their yield are very important for managers to accept the implementation of these courses. In order to investigate the impact of training the trainee's knowledge, skills, and performance, organizations measure training effectiveness. These effects gradually become more visible. The dynamic effects resulting from the training of different SDMUs in an organization can be considered and analyzed in a network structure. The size of human resources, the volume and type of work, salary level, and physical conditions are the inputs of an SDMU, and its outputs include the level of job satisfaction, the level of skill and expertise, and the quality of work.

Fig. 9 shows a network of example 2 that includes three operational units (SDMU1, SDMU2, and SDMU3), the monitoring (M) unit, and also the human resources (HR) unit. In this example, the level of skill and expertise are considered a dynamic effect, which is achieved in two time periods. Table 4 represents the data of example 2. By SSBM-I, the inverse of efficiency is calculated for 30 DMUs. Their efficiency is shown in Table 5. Based on these results, DMU13 and DMU9 have minimum and maximum relative efficiency, respectively.
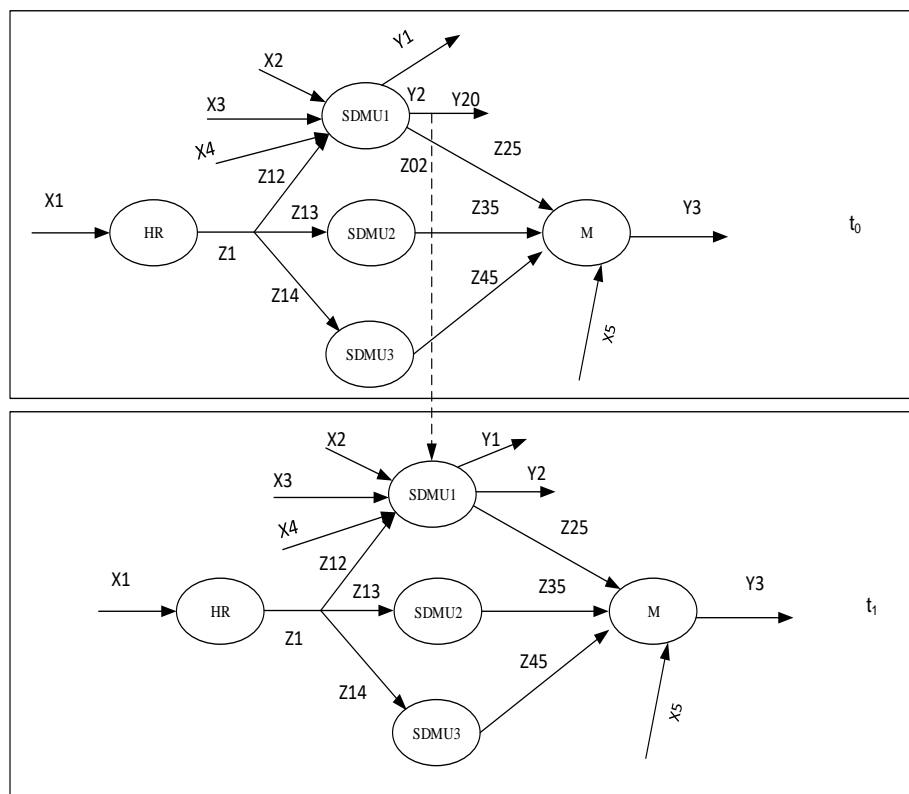

Fig. 9. A network with ordinary links in two time periods.

Table 4. Data related to DMUs of the effectiveness of training courses example.

DMU	α	X1		X2		X3		X4		Z1		Z12		Z13		Z14		Y1		Y2		Y20	Z02	Z25		Z35		Z45		X5		Y3	
		t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_0	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1	t_0	t_1
1	0.88	5	5	2	4	16	32	8	16	20	20	3	3	7	7	10	10	49	69	49	79	43	6	3	6	28	28	19	19	18	18	18	16
2	0.88	4	4	3	5	17	31	9	18	15	15	6	6	8	8	1	1	49	69	40	70	35	5	6	12	28	28	17	17	15	15	17	15
3	0.89	9	9	2	4	16	32	8	16	35	35	2	2	22	22	11	11	45	65	54	84	48	6	4	8	26	26	22	22	18	18	17	15
4	0.87	7	7	4	6	15	30	10	20	27	27	8	8	12	12	7	7	48	68	31	61	27	4	7	14	5	5	30	30	19	19	20	18
5	0.89	6	6	3	5	21	36	8	16	24	24	5	5	15	15	4	4	52	72	65	95	58	7	7	14	8	8	15	15	19	19	17	15
6	0.88	5	5	4	6	19	38	11	22	19	19	6	6	5	5	8	8	53	73	41	71	36	5	6	12	23	23	27	27	20	20	20	18
7	0.89	8	8	4	6	17	34	10	20	31	31	9	9	8	8	14	14	45	65	64	94	57	7	8	16	21	21	9	9	19	19	18	16
8	0.88	5	5	2	4	17	28	8	16	22	22	10	10	6	6	6	6	50	70	50	80	44	6	3	6	15	15	8	8	20	20	15	13
9	0.90	5	5	3	5	17	29	9	18	18	18	9	9	4	4	5	5	46	66	58	88	52	6	7	14	8	8	5	5	16	16	21	19
10	0.89	6	6	5	7	16	30	14	28	23	23	7	7	3	3	13	13	54	74	44	74	39	5	9	18	30	30	5	5	17	17	16	14
11	0.90	7	7	4	6	15	32	10	20	26	26	9	9	9	9	8	8	51	71	48	78	43	5	4	8	30	30	6	6	14	14	17	15
12	0.88	8	8	3	5	18	36	9	18	32	32	14	14	10	10	8	8	49	69	60	90	53	7	5	10	28	28	10	10	20	20	16	14
13	0.90	7	7	5	7	15	30	13	26	25	25	11	11	4	4	10	10	51	71	48	78	43	5	4	8	25	25	24	24	18	18	7	5
14	0.89	8	8	3	5	15	28	9	18	33	33	18	18	3	3	12	12	47	67	63	93	56	7	6	12	7	7	16	16	19	19	18	16
15	0.89	6	6	3	4	15	30	9	18	24	24	4	4	16	16	4	4	50	70	54	84	48	6	7	14	22	22	4	4	17	17	18	16
16	0.88	7	7	3	3	18	32	9	18	28	28	9	9	7	7	12	12	51	71	49	79	43	6	5	10	1	1	12	12	18	18	13	11
17	0.88	8	8	3	5	16	29	8	16	27	27	8	8	8	8	11	11	55	75	41	71	36	5	6	12	3	3	20	20	21	21	17	15
18	0.89	5	5	3	5	16	32	8	16	20	20	7	7	9	9	4	4	43	63	61	91	54	7	5	10	18	18	8	8	19	19	18	16
19	0.88	4	4	4	6	18	36	12	24	16	16	8	8	3	3	5	5	51	71	51	81	45	6	8	16	15	15	15	15	17	17	9	7
20	0.88	4	4	2	4	13	26	6	12	18	18	3	3	2	2	13	13	51	71	52	82	46	6	5	10	17	17	1	1	18	18	17	15
21	0.88	4	4	4	6	18	36	11	22	15	15	1	1	12	12	2	2	50	70	59	89	52	7	7	14	8	8	20	20	17	17	19	17
22	0.89	8	8	2	4	17	30	7	14	29	29	18	18	6	6	5	5	51	71	61	91	54	7	5	10	8	8	25	25	15	15	15	13
23	0.89	5	5	3	5	13	26	9	18	21	21	6	6	9	9	6	6	53	73	57	87	51	6	6	12	2	2	1	1	17	17	16	14
24	0.88	7	7	4	6	17	34	11	22	26	26	3	3	14	14	9	9	53	73	33	63	29	4	7	14	22	22	19	19	17	17	18	16
25	0.89	6	6	5	7	17	37	14	28	24	24	9	9	7	7	8	8	46	66	63	93	56	7	9	18	22	22	20	20	21	21	13	11
26	0.88	5	5	4	6	18	36	13	26	22	22	6	6	6	6	10	10	55	75	40	70	35	5	8	16	11	11	29	29	18	18	17	15
27	0.89	8	8	3	5	16	30	9	18	31	31	10	10	9	9	12	12	47	67	46	76	41	5	6	12	2	2	15	15	19	19	20	18
28	0.88	9	9	3	4	21	38	8	16	34	34	15	15	14	14	5	5	54	74	49	79	43	6	6	12	10	10	8	8	20	20	20	18
29	0.88	7	7	3	3	18	36	10	20	30	30	8	8	13	13	9	9	50	70	41	71	36	5	7	14	21	21	27	27	20	20	19	17
30	0.89	5	5	3	5	17	34	9	18	19	19	2	2	12	12	5	5	51	71	37	67	33	4	5	10	14	14	25	25	19	19	18	16

Table 5. The relative efficiency for all of DMUs by SSBM-I.

DMU	Efficiency	DMU	Efficiency
1	0.8643	16	0.7722
2	0.7564	17	0.7541
3	0.8460	18	0.7819
4	0.6906	19	0.5420
5	0.7955	20	0.9149
6	0.6596	21	0.7692
7	0.7524	22	0.9183
8	0.8150	23	0.8953
9	**<u>1.0000</u>**	24	0.6605
10	0.7524	25	0.6173
11	0.8432	26	0.6631
12	0.7189	27	0.7949
13	**<u>0.4815</u>**	28	0.8460
14	0.9074	29	0.8375
15	0.8764	30	0.6897

The optimal and actual values in two periods t_0 and t_1 are calculated for DMU #13 as the most inefficient DMU in Table 6. The "*" in this table indicates the absence of the intermediate product in the period t_1. In order to achieve full efficiency for DMU #13, the values of the inputs $X1$, $X2$, and $X4$ should be reduced in the period t_0 and $X2$ and $X4$ in period t_1. Besides, $Y1$, $Y2$, and $Y3$ have significant differences with the optimal values in both periods. This difference between the optimal and actual values of the SDMU indicates that the inefficiency started from the first SDMU of DMU #13 and strengthened throughout the network. The value of $Z02$, $Y1$ and $Y2$ in period t_1 indicate that part of this inefficiency has been transferred to SDMU1 through the dynamic link. Therefore, the dynamic effect can be transferred in time periods and included in the calculations.

Table 6. The actual values and the optimal ones for links over two time periods of DMU #13.

	t_0		t_1	
	Actual	Optimal	Actual	Optimal
$X1$	7	3.834	7	7
$X2$	5	3.462	7	5.77
$X3$	15	15.002	30	30.004
$X4$	13	10.386	26	20.772
$Z1$	25	14.91	25	31.5
$Z12$	11	6.924	11	6.924
$Z13$	4	7.352	4	1.929
$Z14$	10	0.636	10	25.909
$Y1$	51	61.162	71	84.242
$Y2$	48	65.778	78	100.40
$Y20$	43	58.968	*	*
$Z02$	5	6.804	*	*
$Z25$	4	6.924	8	13.848
$Z35$	25	19.299	25	19.29
$Z45$	24	6.36	24	3.081
$X5$	18	18	18	18
$Y3$	7	23.009	5	20.647

6. Conclusions

This paper studied the interrelations among networks with dynamic effects and introduced a structure based on these interrelations to measure dynamic effects in different periods easily. Two models based on slack variables were proposed to measure the efficiency of DMUs with dynamic effect structures: SSBM and ASBM. These models measure the overall efficiency of the network for the whole period or separately by considering continuous-time structures and adding virtual DMUs with dynamic effects. In addition, the proposed models can calculate the optimal values of each link including initial input (in an input-oriented state), final output (in an output-oriented state), and intermediate products. In all of them, decision-makers can utilize these models to improve the performance of network structures with dynamic effects. Comparing the organization's performance in different periods and identifying inefficient resources are the main merits of the proposed models.

This paper can be extended by developing a method to determine the consumption optimal value of intermediate products (α) and/or presenting a method to measure the efficiency and effectiveness of networks with dynamic effects simultaneously. Finally, future research may develop a method for the return-to-variable value (VRS).

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