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Integration of the Decisions Associated with Maintenance Management and Process Control for a Series Production System

Hassan Rasay, Mohammad Saber Fallahnezhad*, Yahia Zaremehrjerdi

Faculty of Industrial Engineering, Yazd University, Yazd, Iran

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Abstract

This paper studies a series production system through the integration of the decisions associated with Maintenance Management (MM) and Statistical Process Control (SPC). Hence, the primary question of the paper can be stated as follows: In a series production system, how can the decisions of MM and SPC be coordinated? To this end, an integrated mathematical model of MM and SPC is developed. Using a method of factorial design, sensitivity analyses are performed. According to a stand-alone maintenance model, the effectiveness of the integrated model is assessed. The series production system investigated consists of identical units. Each unit has two operational states including an in-control state and an out-of-control state. The system is in-control if both units of the system operate in the in-control state. On the other hand, the system is out-of-control, if at least one of the units operates in the out-of-control state. The failure mechanism of each unit is based on a random variable with a continuous distribution. The results of analyses clarify that the integrated model improves the profit of the system.

Keywords

Control chart, statistical process control, maintenance management, series system.

* **Corresponding Author, Email:** Fallahnezhad@yazd.ac.ir

Introduction

Maintenance Management (MM) and Statistical Process Control (SPC) play a key role in managing production systems. As discussed by many practitioners and researchers, there are many interdependencies between MM and SPC verifying the study of the integrated models (Liu, Jiang, & Zhang, 2017). This section is organized into three subsections. In the first, second and third subsections, the basic concepts of maintenance management, statistical process control, and series system are introduced, respectively.

Maintenance Management (MM)

MM includes activities that are implemented with the aim of restoring or maintaining a production system in a state that the required functions of the system can be economically performed (Ahmad & Kamaruddin, 2012). Four main objectives are mentioned for MM including 1- ensuring system function (availability, efficiency and product quality), 2- ensuring system life (asset management), 3- ensuring safety, 4- ensuring human well-being (Dekker, 1996). Ding and Kamaruddin (Ding & Kamaruddin, 2015) classified the MM policies into five groups including corrective maintenance, preventive maintenance, CBM or predictive maintenance, autonomous maintenance, and design-out maintenance.

Corrective maintenance is the oldest type of maintenance policy and its actions taken to restore a failure system into operational states (Ahmad & Kamaruddin, 2012). Thus, this policy includes the simple actions that are usually performed after the system completely fails or its function reduces to an unacceptable level. Preventive maintenance policy is a more advanced policy for maintenance planning. In the simplest state, this policy prescribes the maintenance actions at the equal distant intervals irrespective of the system operational state. The aim of a preventive maintenance is to retain a system in the operational state and avoid its complete failure (Ahuja & Khamba, 2008). CBM or predictive maintenance is a modern maintenance policy that its aim is to optimize the performance of preventive maintenance (Alaswad & Xiang, 2017). This policy was introduced in 1975 with the aim of optimizing the performance of preventive maintenance. The basic of the CBM policy is condition monitoring. In condition monitoring, the information about the system operational state is collected, then this information is analyzed and based on these analyses, an appropriate decision about the maintenance actions are taken. In autonomous maintenance, the maintenance and production department cooperate to perform maintenance jobs. Design-out maintenance is a policy that its aim is to improve rather than just conducting maintenance actions (Ding & Kamaruddin, 2015).

Statistical Process Control (SPC)

SPC consists of some problem-solving tools that are effective in reducing process variation and improving process capability and stability. Thus, reducing process variation can be stated as the primary goal of SPC (Montgomery, 2009). SPC includes seven major tools: 1- Histogram or steam-and-leaf plot, 2- check sheet, 3- Pareto chart, 4- cause-and-effect diagram, 5- defect concentration diagram, 6- scatter diagram, and 7- control chart. Elimination of the process variation is the eventual goal of SPC (Montgomery, 2009).

In each production process, two types of variation exist including chance cause of variation and assignable cause of variation. Chance cause of variation is a natural or inherent part of a production process. It is usually cumulative of many small effects, essentially unavoidable. On the other hand, assignable causes of variation are generally larger than chance causes of variation. Three main sources exist for the assignable causes which include operator errors, defective raw material and improperly adjusted machine. A process is in-control if only chance causes affect it. On the other hand, a process is out-of-control if an assignable cause affects the process. Using control charts leads to early detection of the occurrence of the assignable cause, and hence improve the process stability and reduce the process variation.

Series System

Series and parallel systems consisting of several components or units are the classical reparable systems. These systems have received considerable attention in the literature about the reliability and quality engineering (Liu, Yu, Ma, & Tu, 2013). In a series system, failure of each unit leads to the fault of the whole system. In contrast, a parallel system fails while all of its components fail. Reliability of a series system is obtained from the product of the reliability of its component. Thus, the reliability of a series system is less than the reliability of each its units. Figure 1 shows a series and a parallel system.

The rest of the paper is presented as follows: Section 2 is about the problem statement. In Section 3, the importance, aim and the main questions of the research are elaborated. In Section 4, a literature review about the integrated model of SPC and MM is performed. Section 5 develops the integrated model of SPC and MM. In Section 6, the maintenance stand-alone model is derived. Section 7 presents a numerical example and a comparative study of the models. Also, in Section 7, using a design of experiment, the performance of the integrated model is elaborated. Finally, Sections 8 and 9 are dedicated to discussion and conclusion of the paper, respectively.

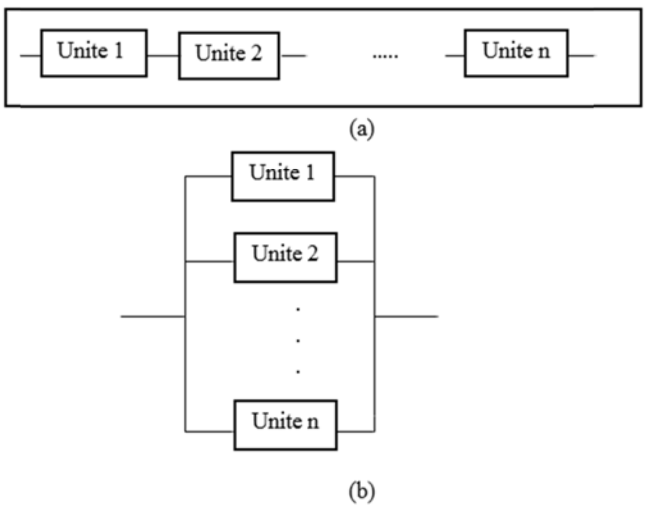


Figure 1. (a) A series production system; (b) a parallel production system

Problem Statement

A series system that has two similar units is studied. Each unit has two operational states: An in-control state that is denoted by State 0, and an out-of-control state that is denoted by State 1. The system is in-control if both units of the system operate in State 0. On the other hand, the system is out-of-control, if at least one unit of the system operates in State 1. The operation of the system in the out-of-control state is undesirable because compared to the in-control state, it leads to much more operational cost and also yields the higher quality costs. For each unit, the time spending in State 0 before a transition to State 1, is considered as a random variable with a continuous distribution that has a general form.

Monitoring the system is conducted as follows: At specific time points such as $(t_1,t_2,\ldots,t_{m-1})$, n units of the items produced by the system are randomly selected and suitable quality characteristic (characteristics) is (are) measured, and a suitable statistic is calculated. This statistic is plotted on the desired control chart. If this statistic falls within the control limits of the control chart, the process will continue its operation without any interruption. If the statistic falls outside the control limits, an alarm is released by the control chart. After that, an investigation is performed on the system to verify this alarm. If this investigation concludes that the chart signal is incorrect, Compensatory Maintenance (CM) is conducted on the system. But if this investigation concludes that the chart signal is correct, Reactive Maintenance (RM) is conducted. In this situation, if both units of the system are in state 1, reactive maintenance of Type 2 (RM (2)) is applied while if only one unit is in State 1, reactive maintenance of Type 1 (RM (1)) is conducted.

Thus, the system is under two types of inspections: The inspections performed based on the sampling from the product at the specific time points such as $(t_1,t_2,\ldots,t_{m-1})$. Henceforth, this inspection is called the sampling inspection. The second type of inspection is conducted at the time that the control chart announces an out-of-control alarm. We call the investigation performed after releasing a signal from the control chart as the maintenance inspection. The sampling inspection is susceptible to a Type I error and Type II error that occur in any control chart in the SPC theory. In a Type I error, it is inferred that the process is out-of-control while it is actually in-control. The probability of this error is usually stated as α. Type II error occurs when the system is out-of-control but the control chart cannot detect this state. The probability of this error is usually denoted as β. In contrast to the sampling inspection, the maintenance inspection can exactly determine the true state of the process. So the maintenance inspection is performed at each of the time points ($t_1,\ldots,t_{m-1}$) if and only if the control chart releases an out-of-control signal.

Different scenarios that may occur during a production cycle are shown in Figure 2. Also, the figure illustrates the structure of the integrated model. This structure is as follows: At each time points of ($t_1,\ldots,t_{m-1}$) that sampling inspection is conducted, if the chart releases an alarm, the maintenance inspection is applied to verify this alarm. If the maintenance inspection specifies the correctness of the alarm, that is, the system is truly announced out-of-control, then Reactive Maintenance (RM) is conducted (Scenarios 5 and 6). In this situation, if both units of the system are in State 1, then reactive maintenance of Type 2 (RM (2)) is implemented, but if only one unit of the system is in State 1, then reactive maintenance of Type 1 (RM (1)) is implemented. The cost and time of RM (2) is more than the cost and time of RM (1). On the other hand, if the maintenance inspection specifies that the alarm released by the control chart is not true, that is, the process is announced out-of-control by mistake, then Compensatory Maintenance (CM) is conducted (Scenario 2).

In addition to the situations explained above, it is also possible the situations in which no alarm is released from the control chart in any of the m-1 inspection time points (scenarios 1, 3, 4). If these scenarios occur, to determine the state of the system at the end of the production cycle, the maintenance inspection is performed on the system in the last period (at time t_m). If this inspection determines that two units are in State 0, Preventive Maintenance (PM) is conducted (Scenario 1). If the maintenance inspection at t_m concludes that each unit of the system is out-of-control, RM is performed on the system (Scenarios 3, 4). RM (1) is applied if only one unit is out-of-control, and RM (2) is applied when both units are out-of-control. Thus, at t_m (at the end of the production cycles) sampling is not conducted, and maintenance inspection is definitely applied. Based on the policy employed, it is clear that maintenance inspection in each production cycle is performed only one time and after that RM, CM or PM is implemented and the process is renewed.

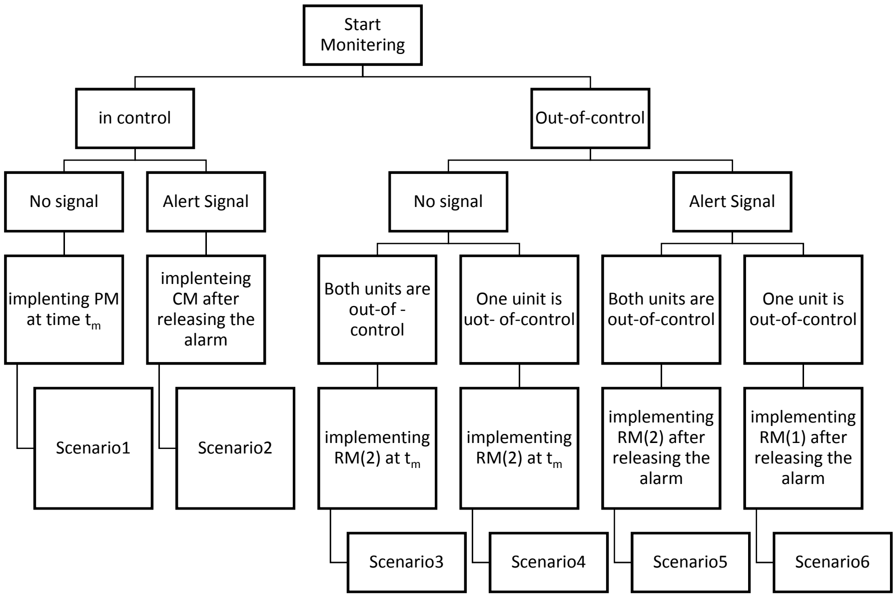


Figure 2. Different scenarios that may occur within a production cycle

Importance, Aim and the Main Questions of the Research

Over the years, as the production systems have shifted from workers to machines, managers have paid increasing attention to the affairs related to maintenance. Automation and mechanization have increased the importance of maintenance management. As a result, the fraction of operational costs associated with maintenance has grown, and the production personnel has reduced (Garg & Deshmukh, 2006). Also, as stated by Wang (2012), next to energy costs, the costs of maintenance can be the largest part of operational costs. This trend leads to increase the role of equipment conditions in controlling the production process and enhancing the product quality. Thus, with the aim of optimizing the profitability of production systems, development of integrated models of MM and SPC has become more and more important.

The aim of this paper is to optimize the profit of the series production system through the integration of the decisions associated with MM and SPC. Hence, the primary question of the paper can be stated as follows: In a series production system, how can the decisions of MM and SPC be coordinated? The second question of the paper is as follows: What is the impact of the system parameters on the decision variables and the objective function of the integrated model? And the last question of the paper is as follows: Compared with the stand-alone model, does the integrated model have a better performance?

To respond to the first question, an integrated model is developed for the system. This model coordinates the decisions of SPC and MM so that the expected profit of the system per time unit (EPT) is maximized. Using a method of experimental design, sensitivity analyses are conducted and the second question of the research is responded. Finally, the last question is responded by developing a stand-alone model and comparing the performance of the integrated model with it.

Experimental design is a statistical method to study systems. In a designed experiment, the input parameters of a system, according to some rules, are systematically changed so that the effect of the changes on the outputs of the system can be observed and analyzed. The input parameters are usually called the factors, and the outputs are called response variables. According to the results of a designed experiment, the effect of the factors on the response variables can be studied. A design includes multiple runs. In each run, the factors are changed and adjusted according to the rules of the design, and then the experiment is conducted and the results are recorded. There are different types of experimental designs such as experiment with a single factor, factorial design, fractional factorial design, and tow-level factorial design (Montgomery, 2013).

Literature Review

In this section, some studies that are closer to the approach of the paper are reviewed. Wan et al. (2018) derived an integrated model of MM and SPC. They applied a synthetic $\overline{X}$ control chart to process monitoring. The effectiveness of the integrated model is demonstrated through comparing it with two stand-alone models. Yang et al. (2009) derived a multi-level maintenance strategy for a production system. The model minimizes the expected cost per time unit through the optimization of replacement age, control limits and two inspection intervals. Liu et al. (2017) according to the geometric process, proposed an integrated model for condition based maintenance and SPC. Zhong and Ma (2017) proposed an integrated SPC and MM model for a two-stage process. They applied a Shewhart control chart and a cause-selecting control chart for monitoring the process. Rasay et al. (2018) developed a mathematical model for integrating maintenance and process control in a multi-stage dependent process. Chi-square control chart is applied in their model. Yin et al. (2015) proposed an integrated model of MM and SPC based on a delayed monitoring. Deterioration of the process is based on a Weibull distribution, and two operational states and a complete failure state are considered for the system. Zhang et al. (2015) employed $\overline{x}$ control chart in the proposed integrated model for SPC and MM. They applied delayed maintenance policy and used Markov chain for modeling of the system.

Naderkhani and Makis (2015) proposed an optimal Bysian control policy to minimize the maintenance costs. The model is characterized by two sampling intervals and two control thresholds. Xiang (2013) proposed an integrated model for a system deterioration while the system has multiple operational states. He employed $\overline{x}$ control chart to monitor the system, and Markov process is applied for modeling of the system. In this model, the impact of preventive maintenance on the system is imperfect so that maintained system restores to a state between the current state and "as-good-as-new state". Liu et al. (2013) studied an integrated model for a series system that has two similar units. They applied $\overline{x}$ chart for monitoring of the process, and deterioration mechanism of each unit of system is described based on an exponential distribution. Due to the exponential assumption about the deterioration mechanism, they apply Markov chain for modeling and analyzing of the system. Also, the system monitoring is conducted at the fixed sampling periods. Panagiotidou and Tagaras (2012) considered a process with two operational states and a failure state. The system has one unit and deterioration mechanism follows a general continuous distribution. Three types of maintenances are applied in the system: Preventive, corrective and minimal maintenance. Preventive and corrective maintenances are considered perfect while minimal maintenance is imperfect.

The novelty of the paper can be stated as development of the model of Liu et al. (2013) in three main directions: (1) Releasing the assumption about the process failure mechanism, hence, no restrictive assumption is applied about the process failure mechanism except that it is continuous with non-decreasing failure rate; (2) this model can be applied for different types of inspection policies such as constant hazard policy or fixed sampling period policy; and (3) the integrated model can be applied for different types of control charts.

Notations and Development of the Integrated Model

First, the notations used in the models of the paper are presented.

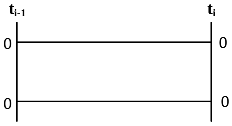
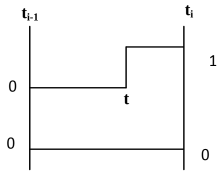
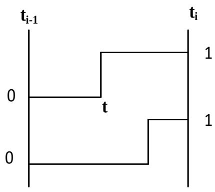
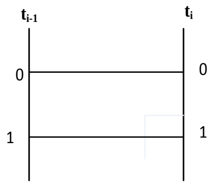
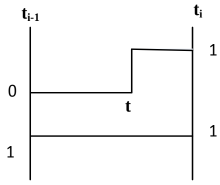
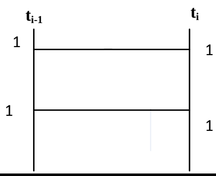
Notation	Description
R_i	Expected net revenue for the operation of the system per time unit when the system is in State i (i=0 system is in-control ,i=1 system is out-of- control; $R_0>R_1$)
W_{QC}	Sampling cost
W_{PM}	The cost of preventive maintenance
$W_{RM(j)}$	The cost of the reactive maintenance of type j (j=1,2)
W_{CM}	Compensatory maintenance cost
W_I	The cost of the maintenance inspection
Z_{PM}	Expected time required for the preventive maintenance
$Z_{RM(j)}$	Expected time required for the reactive maintenance of type j (j=1,2)

Z_{CM}	Expected time required for the compensatory maintenance
Z_I	Expected time required for the maintenance inspection
t	A random variable denoting the time at which the system or one unit of system transits to the out-of-control state
f(t)	Distribution of a random variable representing the time of the quality shift from State 0 to State 1 for each unit of the system
F(t)	Cumulative distribution function (c.d.f) of the time of the quality shift for each unit of the system
g(t)	Distribution of a random variable representing the time of the quality shift from in-control state to out-of - control state for the system
G(t)	Cumulative distribution function (c.d.f) of the time of the quality shift for the system
t_i (i=1,..,m-1)	Time points of sampling inspection (they are the decision variables of the integrated model)
t_m	Time of performing the preventive maintenance (it is the decision variable of the model)
m	Maximum number of inspection periods in each production cycle
n	The size of the samples in each sampling inspection (it is a decision variable of the model)

System Evolution in an Inspection Period

Consider a single arbitrary inspection period such as (t_{i-1},t_i), given the state of each unit of the system, just after the inspection at t_{i-1}, six different scenarios can be considered for the evolution of the system in this period. Table 1 depicts these scenarios, their corresponding probabilities and in-control and out-of-control durations for the system operation in this i period. The scenarios are elaborated in Table 1.

Table 1 .Different Scenarios for the System Evolution During Period (t_{i-1},t_i)

Case	Evolution	Probability of occurrence	Duration of time that the system is out-of-control	Duration of time that the system is in-control
a		$P(a_{t_{i-1}})=\left[\frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})}\right]^2$	0	t_i-t_{i-1}
b		$P(b_{t_{i-1}})=$ $2\frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})}\int_{t_{i-1}}^{t_i}\frac{f(t)dt}{\overline{F}(t_{i-1})}$	t_i-t	t-t_{i-1}
c		$P(c_{t_{i-1}})$ $=1-p(a_{t_{i-1}})-p(b_{t_{i-1}}$	t_i-t	t-t_{i-1}
d		$P(d_{t_{i-1}})=\frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})}$	t_i-t_{i-1}	0
e		$P(e_{t_{i-1}})=1-p(d_{t_{i-1}})$	t_i-t_{i-1}	0
f		$P(f_{t_{i-1}})=1$	t_i-t_{i-1}	0

Case a. In this case, immediately after the inspection at t_{i-1}, both units of the system were operating in-control (State 0), and in the inspection period (t_{i-1},t_i), none of the unit shift were State 1. Hence, both units of the system are also in control at t_i. Thus, provided that the system is in control at t_{i-1} , the system is in-control at t_i, , if and only if, non-units shifts are in State 1 during this interval. Consequently, the probability of evolution of the system based on the case a in the period (t_{i-1},t_i) is computed based on the following conditional probability:

$$P(a_{t_{i-1}}) = P^2\left(t > t_i \mid t > t_{i-1}\right) = \left[\frac{P(t > t_i)}{P(t > t_{i-1})}\right]^2 = \left[\frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})}\right]^2 \tag{1}$$

In Equation 1, $\bar{F}(t)$ is a complement of F(t). In other words, $\bar{F}(t) = 1 - F(t)$.

Case b. The system is in-control at $t_{i-1,}$ because both units are in State 0. In the period (t_{i-1},t_i),nand at time t $(t_{i-1}<t<t_i)$, one of the units transits to State 1 while the other unit continues to operate in State 0, till the end of this period. Hence, at t_i the system is out-of-control. Thus, the evolution probability of the system, based on Scenario b in period (t_{i-1},t_i), is given by:

$$P(b_{t_{i-1}}) = 2P(t > t_i \mid t > t_{i-1}) \times P(t_{i-1} < t < t_i \mid t > t_{i-1}) = 2\frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})}\int_{t_{i-1}}^{t_i}\frac{f(t)dt}{\bar{F}(t_{i-1})} \tag{2}$$

Case c. In this case the system is in-control at $t_{i-1,}$ while it is out-of-control at t_i , because both units are in State 1 at t_i. Given that system was operating in control at t_{i-1}, only three scenarios (case a, b, c) are possible in the period (t_{i-1},t_i). Thus, the probability of the system evolution based on Scenario c during period (t_{i-1},t_i) is as follows:

$$P(c_{t_{i-1}}) = 1 - p(a_{t_{i-1}}) - p(b_{t_{i-1}}) \tag{3}$$

Case d. In this case, the system is out-of-control at t_{i-1} and t_i. Given that at t_{i-1} one unit is in State 0 and the other unit is in State 1, this scenario will occur if and only if the unit that is in-control at t_{i-1}, does not shift to State 1 in period (t_{i-1},t_i). Thus, the probability of system evolution based on Case d in interval (t_{i-1},t_i) is:

$$P(d_{t_{i-1}}) = \frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})} \tag{4}$$

Case e. If one unit of the system was operating in State 1 and the other unit was operating in State 0 at t_{i-1}, only two scenarios are possible in period (t_{i-1},t_i), namely Scenarios d and e. The probability of Case d is derived, consequently, the probability for the evolution of the system operation based on Case e can be computed as:

$$P(\mathrm{e}_{t_{i-1}}) = 1 - p(d_{t_{i-1}}) \tag{5}$$

Case f. If both units were operating in State 1, immediately after inspection at t_{i-1}, they certainly remain in this state till the next inspection time, t_i. Thus, the probability for evolution of system operation based on Case f in period (t_{i-1},t_i) is 1.

It is worth mentioning the following remarks about the system evolution in the period (t_{i-1},t_i) and sampling inspection at t_i. If the system, before inspection at t_i, operates in State 0, after inspection at t_i, system will continue to its in-control operation with the probability of $1-\alpha$, because it is possible that at the inspection of t_i, control chart releases a false alarm with the probability α. After releasing a false alarm, CM is implemented and the system is renewed at t_i.

On the other hand, if the system is out-of-control, just before inspection at t_i, with the probability β , the sampling inspection and control chart cannot release the out-of-control state and therefore the system will continue its operation in the out-of-control state after inspection t_i. Hence, the state of the system after t_i remains out-of-control. Also, when the system is out-of-control, immediately before t_i, the control chart detects this state with the probability $1-\beta$. Consequently, RM is conducted and the system is renewed.

System State at the Start of Each Inspection Period

Suppose that $P_{t_i}^{00}, P_{t_i}^{01}, P_{t_i}^{11}$ be the probabilities that immediately after inspection at t_i, both units operate in State 0, one unit operates in State 0 and the other unit operates in State 1, and both units operate in State 1, respectively. Now, we proceed to the calculation of these probabilities.

$P_{t_i}^{00}$ is given by:

$$p_{t_i}^{00} = \left[\overline{F}(t_i)\right]^2 (1-\alpha)^i \tag{6}$$

Derivation of Equation 6 is as follows: Both units operate in State 0 at t_i, if the time of the shift to State 1 for both units becomes more than t_i , and in the all previous inspection time points, the control chart correctly indicates that the system is in-control. Note that if the system operates in in-control state, the probability that the control chart identifies this state correctly is $1-\alpha$, but if the control chart releases a false alarm, the CM is implemented and system is renewed

$P_{t_i}^{01}$ is calculated based on this recursive formula:

$$p_{t_i}^{01} = \beta\left[p_{t_{i-1}}^{00} \times p(b_{t_{i-1}}) + p_{t_{i-1}}^{01} \times p(\mathrm{d}_{t_{i-1}})\right] \tag{7}$$

This equation is obtained as follows: With respect to Table 1, it is clear that the cases in which, at the end of the period (t_{i-1},t_i), one unit is in-control and the other unit is out-of-control, correspond to Cases b and d. Hence, the sum of terms inside the brackets is the probability of operation of the system in the situation that one unit is in State 0 and the other unit is in State 1, just before the inspection at t_i. Also, if during an interval the system transits to out-of-control state, with the probability of β , the control chart cannot detect this state at t_i, and the system will continue its operation in the out-of-control state. Thus, the two terms inside the square brackets are multiplied by β .

In the similar way that Equation 7 is derived, the following equation is obtained for $P_{t_i}^{11}$:

$$p_{t_i}^{11} = \beta\left[p_{t_{i-1}}^{00} \times p(\mathrm{c}_{t_{i-1}}) + p_{t_{i-1}}^{01} \times p(\mathrm{e}_{t_{i-1}}) + p_{t_{i-1}}^{11} \times 1\right] \tag{8}$$

Because all the maintenance types are assumed perfect, at the start of each production cycle the following equation is held:

$$P_0^{00} = 1, P_0^{01} = 0, P_0^{11} = 0 \tag{9}$$

Expected in-Control and Out-of-Control Time during Each Interval

Let define T_0^i as the expected time that the system operates in State 0 during the period (t_{i-1},t_i), then T_0^i is obtained based on the following equation:

$$T_0^{\,i} = \mathrm{p}_{t_{i-1}}^{00} \times p(a_{t_{i-1}}) \times (t_i - t_{i-1}) + \mathrm{p}_{t_{i-1}}^{00} \times \int_{t_{i-1}}^{t_i} \frac{2f(t)\overline{F}(t)}{\left[\overline{F}(t_{i-1})\right]^2}(t - t_{i-1})dt \quad ,1 \le i \le m \tag{10}$$

With regard to Table 1, derivation of the first term of Equation 10 is simple. Details for deriving the second term are presented in Appendix 1.

Based on the probability of Case a in Table 1, Equation (10) can be rewritten as:

$$T_0^{\,i} = \mathrm{p}_{t_{i-1}}^{00} \times \left[[\frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})}]^2 (t_i - t_{i-1}) + \int_{t_{i-1}}^{t_i} \frac{2f(t)\overline{F}(t)}{\left[\overline{F}(t_{i-1})\right]^2}(t - t_{i-1})dt \right], \qquad 1 \le i \le m \tag{11}$$

If T_1^i is defined as the expected time that the system operates State 1 during the period (t_{i-1},t_i), the following equation is computed:

$$T_1^{\,i} = (\mathrm{p}_{t_{i-1}}^{01} + \mathrm{p}_{t_{i-1}}^{11}) \times (t_i - t_{i-1}) + \mathrm{p}_{t_{i-1}}^{00} \times \int_{t_{i-1}}^{t_i} \frac{2f(t)\overline{F}(t)}{\left[\overline{F}(t_{i-1})\right]^2}(t_i - t)dt, \qquad 1 \le i \le m \tag{12}$$

The first term in Equation 12is easily obtained using Table 1, and derivation of the second term is similar to the derivation of the second term of Equation 10.

Probability of Performing Each Type of Maintenance

Let P_{CM}^i be defined as the probability of performing CM just after the inspection at t_i. The following equation is derived:

$$P_{CM}^{\,i} = \left[\overline{F}(t_i)\right]^2 \times (1-\alpha)^{i-1}\alpha, \quad 1 \le i \le m-1 \tag{13}$$

To obtain this equation, note that CM is implemented on the system at t_i if, first, the time of the shift for both units becomes more than t_i, and second, the control chart truly indicates that the system is in-control in the i-1 previous inspections, and finally control chart release a false alarm at the i_{th} inspection. Note that in the last inspection period there is no CM. Also, in the special case that m=1 the $P_{CM} = 0$

If $P_{RM(j)}^i$ is defined as the probability of conducting reactive maintenance of type j (j=1,2) then it can be computed using the following equation:

$$P_{RM\,(1)}^{i} = (1-\beta)\times\left[P_{t_{i-1}}^{00}\times P(b_{t_{i-1}}) + P_{t_{i-1}}^{01}\times P(d_{t_{i-1}})\right] \qquad 1\le i \le m-1 \tag{14}$$

Equation 14 can be elaborated as follows. At the end of an inspection period (t_{i-1},t_i), RM (1) is conducted on the system, if one unit operates in State 0, and the other unit operates in State 1. Also, referring to Table 1, the cases in which one unit is in State 0 and the other unit is in State 1, correspond to Cases b and d. Also, RM (1) would be implemented at t_i, if the control chart can detect the out-of-control state. Thus, the two terms inside the square brackets are multiplied by $1-\beta$. Sampling inspection is not conducted in the last inspection period, that is at time t_m, thus, $P_{RM(1)}^{m}$ can be computed based on this formula:

$$P_{RM\,(1)}^{\mathrm{m}} = P_{t_{m-1}}^{00}\times P(\mathrm{b}_{\mathrm{t}_{m-1}}) + P_{t_{m-1}}^{01}\times P(\mathrm{d}_{\mathrm{t}_{m-1}}) \tag{15}$$

Also, the following formulas can be obtained for computing $P_{RM\,(2)}^{i}$:

$$P_{RM\,(2)}^{i} = (1-\beta)\times\left[P_{t_{i-1}}^{00}\times P(\mathrm{c}_{t_{i-1}}) + P_{t_{i-1}}^{01}\times P(\mathrm{e}_{t_{i-1}}) + P_{t_{i-1}}^{11}\right],\ 1\le i \le m-1 \tag{16}$$

And in the last inspection period we have:

$$P_{RM\,(2)}^{\mathrm{m}} = P_{t_{m-1}}^{00}\times P(\mathrm{c}_{\mathrm{t}_{m-1}}) + P_{t_{m-1}}^{01}\times P(\mathrm{e}_{\mathrm{t}_{m-1}}) + P_{t_{m-1}}^{11} \tag{17}$$

Since, the states of the system follow Equation 9at the start of each production cycle, for the special case that m=1, the probabilities $P_{RM(1)}, P_{RM(2)}$ are obtained as following:

$$P_{RM\,(1)}^{1} = P(\mathrm{b}_{t_0});\quad P_{RM\,(2)}^{1} = P(c_{t_0}) \tag{18}$$

According to the assumptions explained about the system, a production cycle is terminated by conducting one of the RMs (RM (1) or RM (2)), CM or PM. Hence, the probability of terminating a production cycle due to performance of PM is:

$$P_{PM} = 1-\sum_{i=1}^{m-1}P_{CM}^{i} - \sum_{i=1}^{m}P_{RM\,(1)}^{i} - \sum_{i=1}^{m}P_{RM\,(2)}^{i} \tag{19}$$

Let define P_{QC}^{i} as the probability of conducting sampling at the end of the period (t_{i-1},t_i), then it can be computed based on the following equation:

$$P_{QC}^{i} = P_{t_{i-1}}^{00} + P_{t_{i-1}}^{01} + P_{t_{i-1}}^{11}; \quad 1 \le i \le m-1 \tag{20}$$

Not that, for the last inspection period (at t_m), sampling is not conducted and only maintenance inspection is implemented. Also, in the special case that m=1, $P_{QC}^{1} = 0$.

Expected Profit per Time Unit

The integrated model can be explained according to a renewal reward process. Thus, the expected profit of the system per time unit, EPT, can be described as the ratio of the expected profit of a production cycle, E(P), over the expected duration of a production cycle E(T):

$$EPT = \frac{E[P]}{E[T]} \tag{21}$$

Based on the notations and assumptions introduced so far, E[P] and E[T] in Equation 21 are given by these equations:

$$E[P] = \mathrm{R}_0 \sum_{i=1}^{m} T_0^{\,i} + \mathrm{R}_1 \sum_{i=1}^{m} T_1^{\,i} - \mathrm{W}_{\mathrm{QC}} \sum_{i=1}^{m-1} P_{QC}^{i} - W_{RM(2)} \sum_{i=1}^{m} P_{RM(2)}^{i} - W_{RM(1)} \sum_{i=1}^{m} P_{RM(1)}^{i} - W_{CM} \sum_{i=1}^{m-1} P_{CM}^{i} - W_{PM}\, \mathrm{P}_{PM} - W_I \tag{22}$$

And,

$$E[T] = \sum_{i=1}^{m} T_0^{\,i} + \sum_{i=1}^{m} T_1^{\,i} + Z_{RM(1)} \sum_{i=1}^{m} P_{RM(1)}^{i} + Z_{RM(2)} \sum_{i=1}^{m} P_{RM(2)}^{i} + Z_{CM} \sum_{i=1}^{m-1} P_{CM}^{i} + Z_{PM} P_{PM} + Z_I \tag{23}$$

Maintenance Model

In this model, it is assumed that only maintenance planning is conducted on the system and there is no sampling inspection. The system starts its operation at the zero-age time while it is in-control. After passing t_m time units from the start of operation, the maintenance inspection is conducted on the system. If this inspection denotes that one or two units of the system is in State 1, RM (1) or RM (2) is conducted on the system, respectively. On the other hand, if this inspection indicates that the system still is in-control, preventive maintenance is performed. Note that in this model, if the system shifts to the out-of-control state before t_m, it continues its operation until t_m.

Based on these assumptions and considering the notations of Section 5, the following equations are obtained for E[P] and E[T] in the maintenance model:

$$E[P]=[\overline{F}(t_m)]^2\left[R_0 t_m - W_{PM}\right] \quad \textbf{(24)}$$
$$+\left(1-\left[\overline{F}(t_m)\right]^2\right)\left(R_0\int_0^{t_m} tg(t\,|\,t<t_m)dt + R_1(t_m - \int_0^{t_m} tg(t\,|\,t<t_m)dt\right)$$
$$-W_{RM(1)}\times 2\overline{F}(t_m)F(t_m) - W_{RM(2)}[\mathrm{F(t_m)}]^2 + W_I$$

$$E[\mathrm{T}]=\mathrm{t_m}+Z_{\mathrm{PM}}[\overline{\mathrm{F}}(t_m)]^2 + Z_{RM(1)}\times 2\overline{F}(t_m)F(t_m) + Z_{RM(2)}[\mathrm{F(t_m)}]^2 + Z_I \quad \textbf{(25)}$$

Finally the expected profit per time unit, EPT, for this model is obtained using equation 21.

Numerical Example and Sensitivity Analysis

This section is classified into two subsections. In the first subsection, the application of the models for a specific state is discussed. In the second subsection, a numerical analysis and a comparison study are conducted. Then, using a factorial design, sensitivity analyses are performed.

Application of the Model for a Specific State

For the numerical analysis conducted, it is assumed that $\bar{x}$ control chart is applied for the system monitoring. A single quality characteristic of product denoted as X is used for the system monitoring. If the system operates in the in-control state, it is assumed that X follows a normal distribution with mean μ_0 and standard deviation σ. In the out-of-control state, the mean of X, shifts from μ_0 to $\mu_1 = \mu_0 \pm \delta\sigma$, but this shift does not affect the system standard deviation. δ denotes the magnitude of the shift, and it is assumed to be constant. At the specific time points ($t_1, t_2, \ldots, t_{m-1}$), n units of the produced items are randomly selected as a sample and the quality characteristic of product ,X, is measured. Based on the information obtained from this sample, the mean of the sample, $\bar{x}$, is calculated and plotted on a $\overline{x}$ chart whit the following control limits: $\mu_0 \pm K\frac{\sigma}{\sqrt{n}}$. K denotes the width of the control limits, and it is a decision variable of the integrated model.

If $\phi(\bullet)$ is denoted as the cumulative distribution function (c.d.f) of the normal distribution, it is easy to show that in the $\bar{x}$ control chart, α and β are equal to:

$$\alpha = 2\phi(-k) \quad and \quad \beta = \phi(k - \delta\sqrt{n}) - \phi(-k - \delta\sqrt{n}) \tag{26}$$

From a theoretical point of view, the inspection time points, t_i, can be any arbitrary value. However, in practice, the inspection frequency should be based on a simple rule such that it could be applied in practice. For example, fixed sampling period and constant hazard policy are tow commonly applied rules in practice for determining inspection times. For the numerical analysis of this section, fixed sampling period is applied. It is assumed that the deterioration of each unit in the system follows a Weibull distribution.

Example, Comparison Study and Sensitivity Analyses

The values of the input parameters of an example are shown in Table 2.

Table 2.The Parameters of the Example

R_1	R_0	C_f	C_v	W_{PM}	W_{CM}	$W_{RM(1)}$	$W_{RM(2)}$	W_I	δ
100	500	5	1	1000	700	1500	2000	100	1

Z_{PM}	Z_{CM}	$Z_{RM(1)}$	$Z_{RM(2)}$	Z_I	Z_{QC}	μ
1.5	1	2	3	0.5	0.05	14

C_f and C_v are the fixed and variable sampling costs, respectively. Hence, the sampling inspection cost, W_{QC} for n units is: $C_f+C_v\times n$. The mean of the Weibull distribution, as denoted in Table 2, is 14. The results of optimizing the two models, for different values of shape parameter (v) of Weibull distribution, are illustrated in Table 3 and Figure 3. By discretizing the continuous variables (k,t_1) in the reasonable ranges, it is used as an exhaustive search to determine the decision variables. The programs for optimizing these models are coded in MATLAB software and are available from authors upon request.

Table 3.Result of Optimization of the Example for Different Values of the Shape Parameter in the Weibull Distribution

	v=1		v=2		v=3		v=4	
	IM[1]	MM[2]	IM	MM	IM	MM	IM	MM
EPT	170	124	238	219	267	262	286	286
t_1	1.9	-	2	-	2.9	-	10	-
k	2.9	-	2.5	-	2.4	-	-	-
n	19	-	17	-	15	-	-	-
m	50	-	9	-	5	-	2	
t_m	93.1	13.7	16	9.7	11.6	9.9	10	10.3

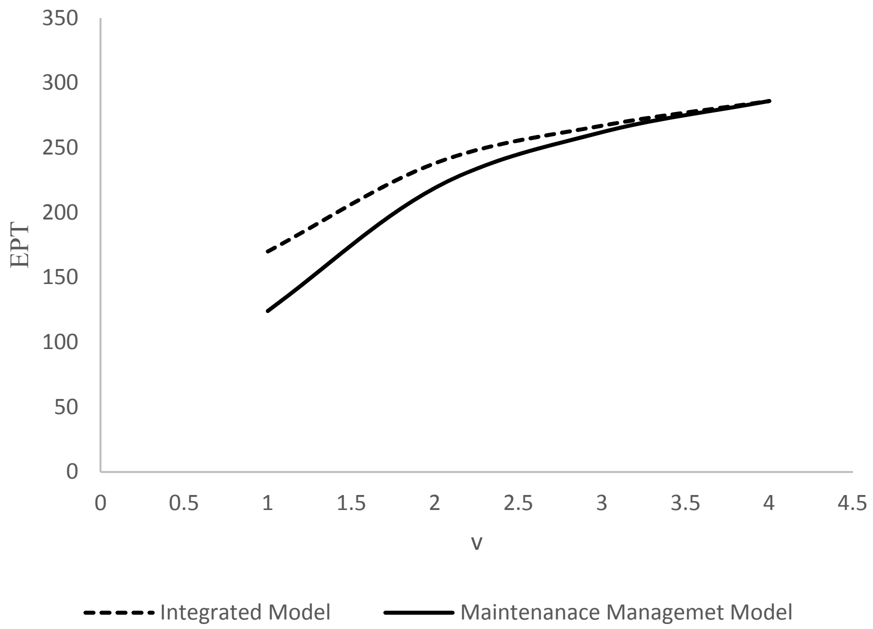


Figure 3. Effect of shape parameter (v) of the Weibull distribution on the values of EPT

In the next step of our analyses, the parameters impacting on the performance of the integrated model are studied. For this purpose, a

1. Integrated model
2. Maintenance model

technique of experimental design, that is Plakett-Burman design, is applied. The integrated model has 17 factors and an experiment with 36 runs is applied. The high and low levels for each factor are illustrated in Table 4.

The results of the experiment design are summarized in Table 5. In this table, the factors are the parameters of the integrated model, and the response variables are the decision variables of the integrated model. The signs in each column indicate the effect of that factor on the decision variable. The positive sign indicates that increasing that factor leads to an increase in the response variable, while the effect of the negative sign is vice versa. The blank entry means that the factor has no meaningful effect on the response variable.

Table 4.The High and Low Level for Each Parameter in the Factorial Design

Factor	δ	V	μ	R_0	R_1	C_f	Cv	W_I	W_{CM}	W_{PM}	$W_{RM(1)}$	$W_{RM(2)}$
Low level(-)	0.5	2	10	300	0	1	0.2	50	300	800	1200	2200
High level(+)	2	4	20	500	100	5	1	100	700	1200	2000	3200

Factor	Z_I	Z_{CM}	Z_{PM}	$Z_{RM(1)}$	$Z_{RM(2)}$
Low level(-)	0	0.5	1	1	2
High level(+)	0.5	1	2	2	3

Table 5. The Results of Factorial Design

Factor / Response variable	δ	v	μ	R_0	R_1	C_v	Wi	W_{CM}	W_{PM}	$W_{RM(1)}$	$W_{RM(2)}$	$Z_{RM(1)}$
EPT		+	+	+			-			-		
t_1				-	+			+				
K	+			-	+			+		-		
n	-			+							-	
m	+	-			-	-			+	-		-
t_m												

Discussion

To coordinate the decision associated with MM and SPC an integrated mathematical model is developed. The model optimally determines the parameters of the control chart and maintenance actions so that the profit of the system would be maximized. Using a technique of experimental design, the effect of each parameter of the system on the decision variables and the objective function is analyzed. The results of these analyses are shown in Table 5. According to the sensitivity analyses, a factor may have an increasing effect on a specific variable, while its effect on another variable is decreasing.

For example, based on Table 5, increasing the value of shape parameter in the Weibull distribution, v, leads to an increase in the value of EPT, while increasing v has a decreasing effect on the value of m. This behavior can be justified because by increasing v in the Weibull distribution, the distribution variance decreases, and consequently prediction of failure time is easier. Some results of Table 5 are intuitive to some extent. For example, increasing the magnitude of the shift, δ_1 , leads to an increase in the value of K and a decrease in the value of n. That is because, for the larger values of the shift, the control chart has more power in detection of the shift. Thus, in the larger value of δ, K becomes larger and n becomes smaller.

The performance of the integrated model is compared with a stand-alone maintenance model with respect to the value of EPT. The results of this comparison are illustrated in Table 3. Accordion to this table, the integrated model leads to a better performance and can improve the profit of the system. As it is shown in Table 3 and Figure 3, for v=1,2,3 the integrated model leads to the larger values of EPT compared to the maintenance model. Also, the difference between the values of EPT of the two models decreases by increasing the value of v, such that in the case v=1 there is the largest difference between EPT in these two models, while the EPT of both models is equal in the v=4. Thus, it is concluded that as the failure time becomes more unpredictable using the integrated model is more conducive. The findings of this section are comparable with the results of the research of Panagiotidou and Tagaras (2012). They reached the similar conclusions about a production system consisting of two operational states and a complete failure state.

Conclusions

A series production system consisting of similar units is investigated. To optimize the profit of the system, the decisions associated with MM and SPC are coordinated through an integrated mathematical model. The model optimally determines the parameters of the control chart and maintenance actions so that the profit of the system can be optimized. Using a technique of factorial design, sensitivity analyses are conducted, and thorough investigation is performed on the model. To evaluate the performance of the integrated model, a stand-alone maintenance model is also presented. Results of the numerical example clarify that, compared with the maintenance model, the integrated model has a better performance. The main novelty of this paper is in two aspects: (1) Development of an integrated model for SPC and MM, while no restrictive assumption is considered about the deterioration mechanism of the units of the system, except that it is continuous with non-decreasing failure rate; and (2) the model can be applied to different types of inspection policies such as constant hazard policy or fixed sampling period policy. Hence, the developed model has a wider application domain with respect to the previous integrated models of MM and SPC.

Access to the real data of the production system is the main limitation of the research. Integration of MM and SPC for more complex systems, development of the models for a system with complete failure state, and application of multivariate control charts for the system monitoring are directions to develop this research.

Appendix 1.

Consider y as a random variable that denotes the time of the shift to the out-of-control state. Process operates in the out-of-control state if at least one of the units operates in the out-of-control state. If the state of the system is in-control at the start of the period (t_{i-1},t_i), then the following equation is derived:

$$\overline{G}(t\,|\,t > t_{i-1}) = \frac{P(y > t, y > t_{i-1})}{P(y > t_{i-1})}$$

$$= \frac{P(y > t)}{P(y > t_{i-1})} = \frac{\left[1 - F(t)\right]^2}{\left[1 - F(t_{i-1})\right]^2} = \frac{[\overline{F}(t)]^2}{[\overline{F}(t_{i-1})]^2}$$

Hence,

$$G(t\,|\,t > t_{i-1}) = 1 - \frac{[\overline{F}(t)]^2}{[\overline{F}(t_{i-1})]^2}$$

Differentiating this equation with respect to t leads to the following equation:

$$g(t\,|\,t > t_{i-1}) = \frac{2f(t)\overline{F}(t)}{\left[\overline{F}(t_{i-1})\right]^2}$$

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