University of Tehran

Iranian Journal of Management Studies (IJMS)

Online ISSN: 2345-3745

Home Page: https://ijms.ut.ac.ir

Developing an All-Unit Quantity Discount Model with Complete and Incomplete Information: A Bertrand Competition Framework

Masume Messi Bidgoli

Department of Industrial Engineering, Golpayegan College of Engineering, Isfahan University of Technology, Golpayegan, Iran. Email: m.messi@iut.ac.ir

ARTICLE INFO

Article type:
Research Article

Article History:
Received 20 September 2021
Revised 11 June 2022
Accepted 21 June 2022
Published Online 28 February 2023

Keywords:
all-unit quantity discount,
Bayesian game,
Bertrand competition,
complete and incomplete information,
inventory control.

ABSTRACT

There are studies regarding monopolistic all-unit quantity discount but we add the duopoly Bertrand competition under incomplete information with it, which is not treated earlier. Then, a systematic procedure is outlined to compute pay-off functions for the developed game. A real numerical example is given in support of the solution procedure and a sensitivity analysis is carried out to examine the important parameters on the equilibrium quantities. This study provides several interesting insights. For instance, it is approved that if retailers know each other completely or not, under some special conditions, rejecting the offered discount is the strictly dominant strategy adopted by the retailers. By doing sensitivity analysis for a real-world case, it is displayed that for maximizing profit, the company should pay more attention to increasing the demand rather than reducing the costs. It could be seen that a change in the value of all parameters except the fixed ordering cost and the holding cost rate results in changing the discount strategy selected by the retailers. Investigating an incomplete information in comparison with the common knowledge case, it is necessary that retailers obtain more accurate knowledge about their rivals.

Cite this article: Messi Bidgoli , M., (2023). Developing an all-unit quantity discount model with complete and incomplete information: A Bertrand competition framework. *Iranian Journal of Management Studies* (*IJMS*), 16 (2), 355-373. DOI: http//doi.org/ 10.22059/ijms.2022.319955.674458

Publisher: University of Tehran Press.
DOI: http//doi.org/10.22059/ijms.2022.319955.674458

1. Introduction

In Economic Order Quantity (EOQ) models, it is assumed that the demand for the product is constant, while demand is a decreasing function of the price. In other words, each retailer is obliged to consider the behavior of his rivals when he determines the price and ordering lot size of the product. Another assumption in EOQ model is that the unit cost of the purchased product is constant. However, this condition is hard to encounter in the real world. It is a common practice that the suppliers offer quantity discount to entice the retailers to purchase more and to achieve economies of scale for transportation and processing costs.

The applicability of various discount models may be limited by the fact that they all ignore competitive reactions to one's actions. To the extent that competitors react to a retailer's actions, these models may be oversimplifications of reality. Imperfect competition, such as oligopolistic competition, is a model that can be applied to a lot of situations. An oligopolistic market consists of a few retailers with a large number of customers, and the retailers have the power to influence the price-demand relationship. Due to the Antitrust Act, many industries in developed countries are oligopolistic, e.g., consumer goods, cars, airline tickets, power. There are two main competitions among the components of supply chains: inter-brand competition and intra-brand competition. In the inter-brand competition, the firms try to develop the differentiated products and their competition is based on brands. Some well-known inter-brand competitions are Coca Cola vs. Pepsi-Cola, Levi vs. GWG jeans, Kellogg's Corn Flakes vs. Nabisco's Bran Flakes. Customers may prefer each of these brands. Intra-brand competition is, on the other hand, competition within a brand. In this case, the distributors or retailers of the same branded product or substitutable products compete against each other. For example, a pair of branded lady's shoes may be sold at a lower price in a low-end shop as compared to a more upmarket shoe shop. Apple stores compete with Wal-Mart stores that sell Apple products. If inter-brand competition is weak, the manufacturer may dedicate some degree of market power to its distributors. The power may give the retailers authority to set their prices and, they could decide on their quantity orders and accepting or rejecting the proposed quantity discounts by the manufacturer. This research investigates these situations where two competitive retailers decide simultaneously on their required products' lot size that must be ordered to a monopolistic supplier with an all-unit quantity discount schedule to motivate the retailers to order in larger than regular order quantities during a limited time. As seen in Figure 1, in this model, each retailer must decide on retailing price and accepting or rejecting the discount offered by the supplier by considering the strategies of its rivals.

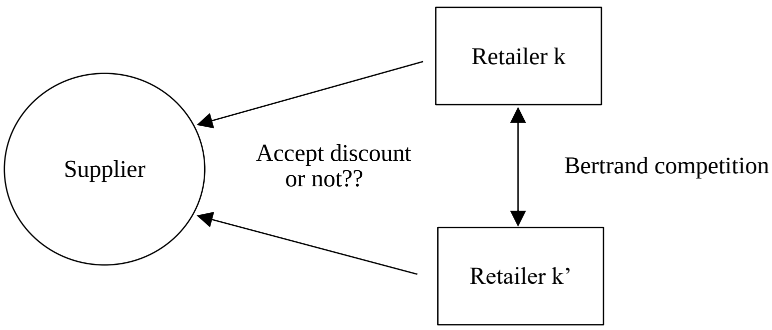

Figure 1. The Relation Between Decision Makers

Because of existing relation between retailers' prices and relation between their demands and their costs, the product price is influenced by the costs of retailers. Thus, each retailer must know its rival's cost structure. Nevertheless, this assumption is hard to encounter in practice and retailers do not have complete information about each other. In this research, we investigate the best decision for each retailer on retailing price and accepting or rejecting offered discount by the supplier for both cases in which a retailer has complete and incomplete information about its competitor's holding cost rate.

The general structure of the present study is as follows. The second section of the study is devoted to literature review. The third section explains all-unit quantity discount model under Bertrand competition with complete and incomplete information. In section 4, a practical example is solved by the proposed model, and a sensitivity analysis for complete information model and benefit analysis for the cases with incomplete information are done for this instance. Finally, in the last section, conclusion is presented along with suggestions for future, more extended studies.

2. Literature Review

2. 1 Literature Review on Monopolistic Discount Models

Discount models have been studied extensively in recent years. Some researchers have paid attention to this problem from buyer perspective and some others have considered buyer-supplier perspective. In the former works, the buyers receive some discount offers that can purchase larger orders of components in lower prices. They should consider the costs and the benefits resulting from these offers and make some reasonable purchasing decisions. In the latter category, a quantity discount is used as a coordination scheme to maximize the joint profit of both the buyer and the supplier. Benton and Park (1996) reviewed the literature on the discount models from both the buyer and the buyer–supplier perspectives.

Some discount models have studied the behavior of only one buyer (or retailer) or some independent retailers, i.e., the market demand for a retailer only is influenced by his own parameters, and the effects of the other retailers' decisions on his demand are disregarded. Shi et al. (2012) studied a joint ordering and pricing model in which the demand linearly depends on the selling price. Their analysis indicated that more profit can be obtained when the supplier provides quantity discount. An inventory model with stock-dependent demand and temporary promotional quantity discounts was formulated and the effect of quantity discounts offered by the supplier on the retailer's ordering policy was analyzed by Shah (2014).

Alfares and Ghaithan (2016) developed an all-unit quantity discount model for a price-dependent demand and a time varying holding cost. The results showed that the profit is mostly affected by the demand's parameters. This means that companies should increase their customer demands and then they should reduce their purchasing and ordering costs to improve their profits. In addition, they showed that decreasing the selling price is the third way to increasing the total revenues.

It is good to mention that all of these works have been studied under a monopoly situation, assuming there is no rival to be considered in inventory decisions. Only a few researchers have considered competition among retailers.

2. 2 Literature Review on Competitive Inventory Models

There are two main types of competition known as Bertrand and Cournot competition. In Bertrand model, the retailers choose how much to charge for their products, but in Cournot model the retailers compete via allocations and not prices.

Some of the works in studying competitive inventory models are reviewed in this section. Melnikov (2017) studied a Bertrand's competition in the retail duopoly with asymmetric costs. It was found in this research that the level of profitability of logistics costs can be planned by the retailers and also the level of the entry barrier to the market can be assessed by them. Saha et al. (2021) investigated a retailer and two upstream manufacturers in a two-period horizon in which retailer decided how much inventory must hold and manufacturers determined their equilibrium prices in a Bertrand competition. They showed cooperative manufacturers can obtain higher profit if there is not strategic inventory. In addition, it was shown that under the Bertrand competition, a commitment contact can outperform the decision. Mahmoodi (2019) studied a duopoly competition where prices of deteriorating products and their replenishment cycles must be determined. The main insight of this research was that the retailer with larger market suffers from competition but the retailer with smaller market obtains more profit from competition. Mahmoodi (2020) used bi-level programming to model in-chain, and chain-to-chain duopoly competition for two competing supply chains. The retailer's profit was found to be influenced by the market size more than the manufacturers. In addition, for the symmetrical problem, the more the competition intensity increased, the more the consumer surplus and the manufacturers' profit were earned. For the nonsymmetrical problem, the more the competition intensity increased, the less profit was obtained by the manufacturer with the bigger market size and the more profit was obtained by the manufacturer with the smaller market size.

2. 3 Literature Review on Competitive All-Unit Quantity Discount Models

Among competitive inventory models, there are a few works that have studied an all-unit quantity discount model. Xiao and Qi (2008) studied a Bertrand game between two retailers with complete information and investigated whether the menu of two-part tariffs as well as the all-unit quantity discount scheme coordinate a supply chain. They showed that if the costs of two retailers have a remarkable difference, then the all-unit quantity discount scheme cannot coordinate the supply chain with disruptions. While the cost disruption may affect the wholesale prices, order quantities, and retail prices, it is optimal for the supply chain to keep the original coordination mechanism if the production cost change is sufficiently small.

Nearest work to ours has been done by Navidi and Bidgoli (2011). They presented a Cournot competition for an all-unit quantity discount problem with complete and incomplete information. They investigated the cases with two retailers competing via their optimal order quantities. They approved for their model with complete information that the strictly dominant strategy for both retailers are accepting discount.

The key feature differentiating our paper from Navidi and Bidgoli (2011) is that, in this paper, an all-unit quantity discount model under a Bertrand competition is studied. Due to existing competition between retailers, their costs affect their products' prices. Thus, the cost structure of the competitors must be considered by each retailer. However, in practice, this assumption is hard to encounter, and the competitors do not know each other completely. In this research, we first consider competitors' holding cost rates known for two retailers and then consider this parameter as their private information. In both cases, we investigate the best ordering lot size, retailing price, and decision on accepting or rejecting the offered discount for each retailer. A strategic game could be used to model the interaction between the retailers.

According to the authors' knowledge, none of the existing studies in the inventory models has simultaneously taken all of these features into consideration. This research deals with an all-unit quantity discount model under a Bertrand competition between retailers in which demand for each retailer linearly depends on its price.

3. All-Unit Quantity Discount Model Under Bertrand Competition with Complete Information

In this section, two key system components in our model framework, namely demand and inventory cost functions, are described. Before that, it is necessary to define some notations.

$\boldsymbol{k}$	Index for two retailers $\boldsymbol{k = A, B}$
A_k	Retailer k's ordering cost per order
H	Retailers' holding costs per unit of inventory per selling season
μ_k	Holding cost rate of retailer k
q_k	Ordering lot size of retailer k
TC_k	Total annual inventory cost for retailer k
c_{1k}	Price paid per purchased unit, if retailer k does not accept the suggested discount
c_{2k}	Price paid per purchased unit, if retailer k accepts the suggested discount
Q_0	Break point of discount
p_k	Retailer k's retailing price (decision variable)
D_k	Demand for retailer k's channel
α_k	Market base for retailer k
β	Marginal demand changed by channel price
γ	Migration rate for the perceived price difference
π_k	Profit function of retailer k

In our model, there is only one supplier that provides the product of retailers in the following price schedule.

$$\begin{cases} c_{1k} & ,q_k \leq Q_0 \\ c_{2k} & ,Q_0 < q_k \end{cases} \tag{2-0}$$

According to the above schedule, if the retailer k $(k = A, B)$ does not accept the suggested discount and the orders are less than Q_0 , he/she must pay c_{1k} per purchased unit, otherwise c_{2k} must be paid such that $c_{2k} < c_{1k}$.

Then retailer k decides on his ordering quantity q_k . We assume that the supplier is obliged to meet the retailers' orders and has enough capacity.

For each retailer, we adopt linear model structure for demand function. It is worth mentioning that these functions are widely used in supply chain due to their tractability and they have been empirically tested by some researchers such as Mahmoodi (2019). We use the same linear structure for demand function. If nonlinear demand functions can be approximated by linear ones, our managerial insights still hold.

$$D_k = \alpha_k - \beta p_k + \gamma\left(p_{k'} - p_k\right), \left(k, k' = A, B \ , k' \neq k\right) \tag{2-1}$$

γ is migration rate for the perceived price difference. It means that if the prices of two retailers differentiate and customers become aware of this difference, they will buy from the other retailer at the rate of γ. It forms a horizontal competition between these retailers. In addition, the following assumptions are made about the parameters in the demand functions:

(1) As seen in real life cases, in comparison with the other market parameters, market base α_k $(k = A, B)$ is large enough.

(2) We assume $\beta > \gamma$, which means that the direct price has greater effect on the demand than the price difference between the two retailers.

In these competitive cases, each retailer tries to propose a retailing price that maximizes its profit. We consider a classical single period problem (SPP) that deals with the purchasing inventory problem for single-period products, such as perishable, seasonal goods and products with short life cycles (e.g., fashion clothes and electronic products, as studied by Zhang et al. (2009) and Forghani et al. (2013)). In SPP, only one purchase order is allowed and the ordered lot is placed at the beginning of the period. Total purchased quantities are sold before the end of period (Li et al., 2013). Therefore, there are no remained inventories in the retailer's warehouses at the end of sale season.

3.1 Inventory Cost Function

The inventory holding cost for retailer k may also be computed as a fraction (μ_k) of a unit cost (c_{ik}). Then the holding cost is given by $\mu_k c_{ik}$ for this retailer.

Given a lot size of $q_k = D_k$, we have an average inventory of $D_k/2$. The holding cost per period is thus the cost of holding $D_k/2$ units in inventory for one season and equals to $(D_k/2)\mu_k c_{ik}$. In addition, material cost is considered as $c_{ik}D_k$

By gathering up the mentioned costs, the total inventory cost for retailer k is given as

$$TC_k = c_{1k} D_k + A_k + \left(D_k / 2\right)\mu_k c_{1k} \, , \ i = 1,2 \ , k = A, B \tag{2-2}$$

If $D_k < Q_0$, otherwise the total inventory cost for retailer k is given as

$$TC_k = c_{2k} D_k + A_k + \left(D_k / 2\right)\mu_k c_{2k} \, , \ k = A, B \tag{2-3}$$

3.2 Profit Function of the Retailers

By subtracting inventory costs from revenue for each retailer, we can obtain its profit function as follows.

$$\pi_k = p_k D_k - \left[c_{ik} D_k + A_k + \left(D_k / 2\right)\mu_k c_{ik}\right], i = 1,2 \ , k = A, B \tag{2-4}$$

By substituting D_k in the above function, we have

$$\pi_k = \left[\alpha_k - \beta p_k + \gamma\left(p_{k'} - p_k\right)\right]\left[p_k - c_{ik} - \left(\mu_k c_{ik} / 2\right)\right] - A_k \tag{2-5}$$

when all cost parameters of both retailers are common knowledge, a two-person game with complete information occurs as follows:

Players: retailer (A, B)

Strategy (S_k): {not accept the offered discount (buying at a price of c_{1k}), accept the offered discount (buying at a price of c_{2k})}, $k = A, B$

Payoff function: the profit of each retailer is its payoff that is obtained from relation $(2 - 5)$ for each retailer. All possible situations are represented in Table 1.

Table 1. Payoff Matrix for Two Retailers

Retailer B / Retailer A	**Rejecting the discount (Buying at a price of c_{1B})**	**Accepting the discount (Buying at a price of c_{2B})**
Rejecting the discount (Buying at a price of c_{1A})	$\pi_A(c_{1A},c_{1B}),\pi_B(c_{1A},c_{1B})$	$\pi_A(c_{1A},c_{2B}),\pi_B(c_{1A},c_{2B})$
Accepting the discount (Buying at a price of c_{2A})	$\pi_A(c_{2A},c_{1B}),\pi_B(c_{2A},c_{1B})$	$\pi_A(c_{2A},c_{2B}),\pi_B(c_{2A},c_{2B})$

From the first-order condition (FOC), we have the optimal response function for retailer k:

$$\max_{p_k}\left\{\left[\alpha_k-\beta p_k+\gamma\left(p_{k'}-p_k\right)\right]\left[p_k-c_{ik}-\left(\mu_k c_{ik}/2\right)\right]-A_k\right\}$$

$$\frac{\partial\pi_k}{\partial p_k}=0\Rightarrow p^*_k=\frac{\alpha_k+\gamma p_{k'}}{2(\beta+\gamma)}+\frac{c_{ik}}{4}\left(2+\mu_k\right),\ (k'=A,B\ ,k'\neq k) \tag{2-6}$$

Similarly, the optimal price for the other retailer could be found.

As seen in relation (2-6), the best price selected by each retailer depends on that of the other retailer. Therefore, by substituting $p^*_{k'}$ in p^*_k, we have

$$p^*_k=\frac{2\left(\beta+\gamma\right)\left(\alpha_k+\frac{c_{ik'}}{4}\gamma\left(2+\mu_{k'}\right)\right)+\gamma\alpha_{k'}+\left(\beta+\gamma\right)^2c_{ik}\left(2+\mu_k\right)}{4\left(\beta+\gamma\right)^2-\gamma^2},(k,k'=A,B,k'\neq k) \tag{2-7}$$

To suggest the optimal retailers' prices, it is desired to determine whether they accept the offered discount or not. To this end, the price schedule offered by the supplier to retailer k is changed by substituting D_k from relation (2-1) into relation (2-0) as follows:

$$\begin{cases}c_{1k} & ,D_k\leq Q_0\\ c_{2k} & ,D_k>Q_0\end{cases}\Rightarrow\begin{cases}c_{1k} & ,p_k\geq\dfrac{\alpha_k-Q_0+\gamma p_{k'}}{\left(\beta+\gamma\right)}=p_k^o\\ c_{2k} & ,p_k<p_k^o\end{cases} \tag{2-8}$$

From now on, the first inequality ($p_k\geq p_k^o$) is named rejection domain and the second inequality as acceptance domain.

In continue, lemma 1 is presented to show that each retailer prefers to reject the discount under some conditions.

<u>**Lemma 1**</u>. When a monopolistic supplier offers an all-unit quantity discount to two competitive retailers, if both retailers' optimum prices set in their related domains, i.e., $p^*_k\left(c_{1k},c_{jk'}\right)\geq p_k^o$, and $p^*_k\left(c_{2k},c_{jk'}\right)\leq p_k^o,\forall j=1,2\ ,\ \ k,k'=A,B\ \ ,k'\neq k$, then we have for retailer k:

$$\pi_k\left(c_{2k},c_{ik'}\right)\leq\pi_k\left(c_{1k},c_{ik'}\right),\forall i=1,2,k,k'=A,B,k'\neq k \tag{2-9}$$

In other words, rejecting the offered discount is the strictly dominant strategy of retailer k.

Proof (lemma 1): By replacing p_k with p^*_k in (2-5), we do as follow:

$$\begin{aligned}&\pi_k\left(c_{2k},c_{ik'}\right)-\pi_k\left(c_{1k},c_{ik'}\right)=\\&\frac{\left(\beta+\gamma\right)\left(2+\mu_k\right)\left(c_{2k}-c_{1k}\right)}{B}\{\left(\alpha_k+A_2/B\right)\left(\beta+\gamma\right)+\\&\left(1-\frac{2\left(\beta+\gamma\right)^2-\gamma}{B}\right)(1/2)[A_1+\left(\beta+\gamma\right)^2\left(2+\mu_k\right)\left(c_{2k}+c_{1k}\right)]\}\quad(2-10)\end{aligned} \tag{2-10}$$

where

$$A_1=2\left(\beta+\gamma\right)\left(\alpha_k+\frac{c_{ik'}}{4}\gamma\left(2+\mu_{k'}\right)\right)+\gamma\alpha_{k'}, \tag{2-11}$$

$$A_2=\left(\beta+\gamma\right)^2c_{ik'}\left(2+\mu_{k'}\right)+\gamma\alpha_k+2\left(\beta+\gamma\right)\alpha_{k'}, \tag{2-12}$$

$$B = 4(\beta+\gamma)^2 - \gamma^2, \tag{2-13}$$

Since in the discount schedule we have $c_{2k} < c_{1k}$, then $c_{2k} - c_{1k} < 0$. On the other hand, it is clear that $B > 0$. This means that $\frac{(\beta+\gamma)(2+\mu_k)(c_{2k}-c_{1k})}{B} < 0$. If the second part of relation (2-10) is equal or greater than 0, then we can conclude $\pi_k(c_{2k}, c_{ik'}) - \pi_k(c_{1k}, c_{ik'}) \leq 0$. It is clear that $\left(1 - \frac{2(\beta+\gamma)^2-\gamma}{B}\right) > 0$ for all quantities of β and γ . In this case, we have for every $(c_{ik}, c_{jk'})$,

$$\pi_k\left(c_{2k}, c_{ik'}\right) \leq \pi_k\left(c_{1k}, c_{ik'}\right), \quad \forall i = 1,2, \;\; k, k' = A, B \;, k' \neq k \tag{2-14}$$

The above relation is obtained by replacing p_k with p_k^*. This replacement is correct if each p_k^* sets in its related domains. However, for the other cases, the optimum prices may be found more complicatedly. For this situation, the following algorithm can help compute pay-off matrix, systematically.

Step 1: First, compute $p^*_k(c_{ik}, c_{jk'}), \forall i, j = 1,2\,,\;\; k, k' = A, B\;\;, k' \neq k$. If for every $(c_{ik}, c_{jk'})$, the price of each retailer is set in its related domain, based on lemma 1, the strictly dominant strategy for each retailer can be obtained.

Step 2: If the prices obtained for at least one of the retailers are not in their right domains, for those quantities that do not set in its related spans, in the profit function we substitute the optimal price of that retailer with p_k^o (this is because the profit is a quadratic function of price and when optimal price is out of the domain, the maximum profit is achieved at the minimum price). This fact is displayed in Figures 2 and 3.

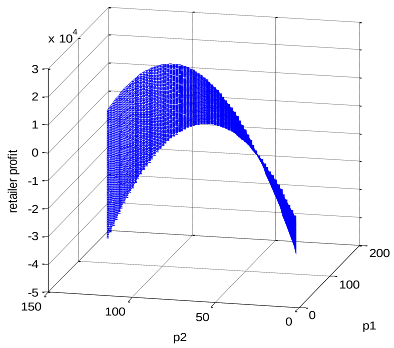

Figure 2. Profit Function of Retailer **1**

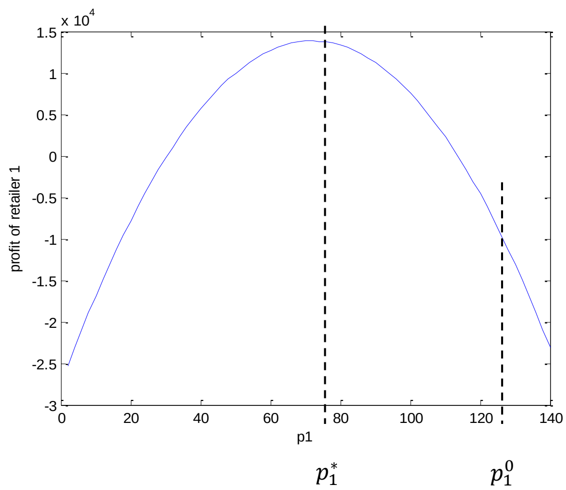

Figure 3. Profit Function of Retailer **1** for the Case $\boldsymbol{p_1^*} \leq \boldsymbol{p_1^0}$ (for a Constant Value for $\boldsymbol{p_2^*}$)

For instance, consider (c_{1A}, c_{1B}). After computing the optimal prices, if only $p^*{}_B(c_{1A}, c_{1B})$ does not set in its domain for (c_{1A}, c_{1B}) i.e., $p^*{}_A(c_{1A}, c_{1B}) \geq p_A^o(c_{1A}, c_{1B})$ and $p^*{}_B(c_{1A}, c_{1B}) < p_B^o(c_{1A}, c_{1B})$, then its profit is computed by relations (2-15).

$$\pi_B(c_{1A}, c_{1B}) = \left[\alpha_B - (\beta+\gamma)p_B^o + \gamma p^*{}_A\right]\left[p_B^o - c_{1B} - (\mu_B c_{1B}/2)\right] - A_B, \tag{2-15}$$

On the other hand, in this case we must compute $p^*{}_A(c_{1A}, c_{1B})$ again as (2-17).

$$p^*{}_A = \frac{\alpha_A + \gamma p_B^o}{2(\beta+\gamma)} + \frac{c_{1A}}{4}(2+\mu_A), p_B^o = \frac{\alpha_B - Q_0 + \gamma p^*{}_A}{(\beta+\gamma)} \tag{2-16}$$

$$\Rightarrow p^*{}_A = \frac{\alpha_A(\beta+\gamma) + \gamma\alpha_B - \gamma Q_0 + (\beta+\gamma)^2(2+\mu_A)(c_{1A}/2)}{2(\beta+\gamma)^2 - \gamma^2} \tag{2-17}$$

If this price sets in its rejection domain i.e., $p^*{}_A(c_{1A}, c_{1B}) > p_A^o$, we have

$$\pi_A(c_{1A}, c_{1B}) = \left[\alpha_A - (\beta+\gamma)p^*{}_A + \gamma p_B^o\right]\left[p^*{}_A - c_{1A} - (\mu_A c_{1A}/2)\right] - A_A \tag{2-18}$$

Otherwise, we use p_A^o and p_B^o and compute its profit by (2-19).

$$\pi_A(c_{1A}, c_{1B}) = Q_0\left[p_A^o - c_{1A} - (\mu_A c_{1A}/2)\right] - A_A \tag{2-19}$$

where $p_A^o = \frac{(\alpha_A - Q_0)(\beta^2+2\beta\gamma) + \gamma[(\alpha_B - Q_0)(\beta+\gamma) + \gamma(\alpha_A - Q_0)]}{(\beta+\gamma)(\beta^2+2\beta\gamma)}$.

For example, for (c_{1A}, c_{1B}), if neither $p_A(c_{1A}, c_{1B})$ nor $p_B(c_{1A}, c_{1B})$ set in their domain i.e., $p_A(c_{1A}, c_{1B}) < p_A^o(c_{1A}, c_{1B})$ and $p_B(c_{1A}, c_{1B}) < p_B^o(c_{1A}, c_{1B})$, then the retailers' profits are as follows.

$$\pi_A(c_{1A}, c_{1B}) = \left[\alpha_A - (\beta+\gamma)p_A^o + \gamma p^0{}_B\right]\left[p_A^o - c_{1A} - (\mu_A c_{1A}/2)\right] - A_A \tag{2-20}$$

$$\pi_B(c_{1A}, c_{1B}) = \left[\alpha_B - (\beta+\gamma)p_B^o + \gamma p^0{}_A\right]\left[p_B^o - c_{1B} - (\mu_B c_{1B}/2)\right] - A_B \tag{2-21}$$

where $p^0{}_A(c_{1A}, c_{1B})$ and $p^0{}_B(c_{1A}, c_{1B})$ are computed as follows:

$$p_A^o = \frac{\alpha_A - Q_0 + \gamma p^0{}_B}{(\beta+\gamma)}, p_B^o = \frac{\alpha_B - Q_0 + \gamma p^0{}_A}{(\beta+\gamma)} \Rightarrow p^0{}_A = \frac{\alpha_A(\beta+\gamma) + \gamma\alpha_B - (\beta+2\gamma)Q_0}{(\beta+\gamma)^2 - \gamma^2}, p^0{}_B = \frac{\alpha_B(\beta+\gamma) + \gamma\alpha_A - (\beta+2\gamma)Q_0}{(\beta+\gamma)^2 - \gamma^2}$$

The other cases are computed in a similar manner.

Step 3: Now we can represent this game like the one displayed in Table 1. Therefore, the best strategy for each player against its rival could be computed for this matrix.

The flowchart of this decision-making process is displayed in Figure 4.

So far, players know each other completely. In the next section, the cases are studied where the retailers do not know each other completely.

4. All-Unit Quantity Discount Model Under Bertrand Competition with Incomplete Information

Up to this point, the parameters of the model are assumed to be non-private and two players have the same perception about them, but in real situation there are some private parameters. In these situations, ignoring this assumption is oversimplification and the obtained results could not match real cases. If the holding cost rate of each retailer is his private knowledge, then we coincide with a strategic game with incomplete information. A Bayesian game formulation provides a framework for each retailer to decide about its best strategies based on its belief on the type of the rival.

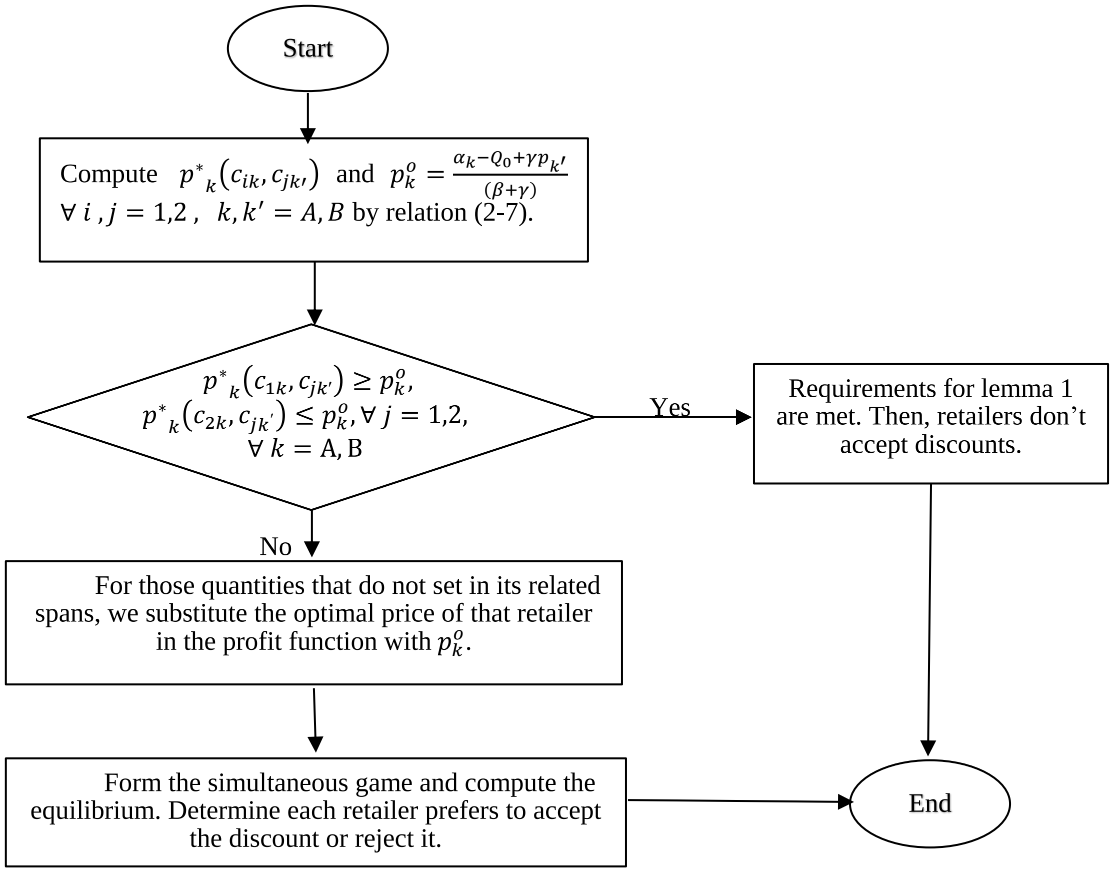

Figure 4. Decision Making Process for the Considered Competitive All-Unit Quantity Discount With Complete Information

However, the holding cost rate of each player is not common knowledge; each retailer has a probability distribution based on its competitor's parameter. For more simplicity, we assume that $\mu_k, (k = A, B)$ is uniformly distributed, i.e., $\mu_k \sim U[\mu_1, \mu_2]$ and two retailers know this. From the first-order condition (FOC), we have the optimal response function for retailer k:

$$p^*_k = \frac{\alpha_k + \gamma p_{k'}}{2(\beta+\gamma)} + \frac{c_{ik}}{4}(2+\mu_k), \ \forall i = 1,2, \ k, k' = A, B, k' \neq k \tag{3-1}$$

Since retailer k has no complete knowledge about $p_{k'}$ thereby the retailer k must predict $p_{k'}$.

$$p^*_{k'} = \int_{\mu_1}^{\mu_2} p^*_{k'} f(\mu_{k'}) d\mu_{k'} = \frac{1}{2(\mu_2 - \mu_1)} \int_{\mu_1}^{\mu_2} \left[\frac{\alpha_{k'} + \gamma p_k}{(\beta+\gamma)} + \frac{c_{ik'}}{2}(2+\mu_{k'}) \right] d\mu_{k'} \tag{3-2}$$

$$p^*_{k'} = \frac{1}{2}\left[\frac{\alpha_{k'} + \gamma p_k}{(\beta+\gamma)} + c_{ik'} \right] + \frac{c_{ik'}(\mu_2 + \mu_1)}{8} \tag{3-3}$$

By substituting (3-3) in (3-1), we obtain the optimal p^*_k:

$$p^*_k = \frac{\beta+\gamma}{4(\beta+\gamma)^2 - \gamma^2} \left\{ 2\alpha_k + \gamma\left[\frac{\alpha_{k'}}{\beta+\gamma} + \frac{4+\mu_2+\mu_1}{4} c_{ik'} \right] + (\beta+\gamma) c_{jk}(2+\mu_k) \right\} \ \forall i, j = 1,2, \ k, k' = A, B, k' \neq k \tag{3-4}$$

To calculate the profit functions for retailer k, first we compute p^*_k and $\widehat{p^*_{k'}}$ for every ($c_{jk}, c_{ik'}$), and investigate if these quantities set in their specified spans in the discount schedule. If the responses are positive for both prices, we set them in $\pi_k(c_{jk}, c_{ik'})$.

$$\pi_k(c_{jk}, c_{ik'}) = \left[\alpha_k - \beta p^*_k + \gamma\left(p^*_{k'} - p^*_k \right) \right]\left[p^*_k - c_{jk} - (\mu_k c_{jk}/2) \right] - A_k \ \forall i, j = 1,2, \ k, k' = A, B, k' \neq k \tag{3-5}$$

In this situation, three cases may happen for every $(c_{jk}, c_{ik'})$, $\forall i,j = 1,2\ ,\ \ k,k' = A,B\ \ ,k' \neq k$:

Case 1- only $p^*{}_k$ does not set in its domain. In this case, substitute $p^*{}_k = p_k^o$ and obtain $\widehat{p^*{}_{k'}}$ from Eq. $(3-3)$, as follows.

$$p_{k'}{}^* = \frac{1}{2}\left[\frac{\alpha_{k'} + \gamma p_k}{(\beta+\gamma)} + c_{ik'}\right] + \frac{c_{ik'}(\mu_2+\mu_1)}{8}, \tag{3-6}$$

$$p_k^o = \frac{\alpha_k - Q_0 + \gamma p^*{}_{k'}}{(\beta+\gamma)} \tag{3-7}$$

$$p_{k'}{}^* = \frac{4(\beta+\gamma)\alpha_{k'} + 4\gamma[\alpha_k - Q_0] + (\beta+\gamma)^2 c_{ik'}[4+\mu_2+\mu_1]}{8(\beta+\gamma)^2 - 4\gamma^2}, \tag{3-8}$$

$\forall i = 1,2,\ k,k' = A,B\ \ ,k' \neq k$

If $\widehat{p^*{}_{k'}}$ does not set in its new domain, substitute $\widehat{p^*{}_{k'}} = p_{k'}^o$.

Case 2- only $\widehat{p^*{}_{k'}}$ does not set in its domain. In this case, substitute $\widehat{p^*{}_{k'}} = p_{k'}^o$ and compute $p^*{}_k$ by Eq. $(3-1)$ as follows.

$$p^*{}_k = \frac{\alpha_k + \gamma p_{k'}^o}{2(\beta+\gamma)} + \frac{c_{jk}}{4}(2+\mu_k),\ \ p_{k'}^o = \frac{\alpha_{k'} - Q_0 + \gamma p^*{}_k}{(\beta+\gamma)} \tag{3-9}$$

$$\Rightarrow p^*{}_k = \frac{\alpha_k(\beta+\gamma) + \gamma\alpha_{k'} - \gamma Q_0 + (\beta+\gamma)^2(2+\mu_k)\frac{c_{jk}}{2}}{2(\beta+\gamma)^2 - \gamma^2} \tag{3-10}$$

Case 3- Neither $p^*{}_k$ nor $\widehat{p^*{}_{k'}}$ set in their domains. In this case, substitute $p^*{}_k = p_k^o$ and $\widehat{p_{k'}{}^*} = p_k^o$.

$$p_k^o = \frac{\alpha_k - Q_0 + \gamma p_{k'}^o}{(\beta+\gamma)},\ p_{k'}^o = \frac{\alpha_{k'} - Q_0 + \gamma p_k^o}{(\beta+\gamma)} \tag{3-11}$$

$$p^*{}_k = p_k^o = \frac{(\beta+\gamma)(\alpha_k - Q_0) + \gamma(\alpha_{k'} - Q_0)}{(\beta+\gamma)^2 - \gamma^2} \tag{3-12}$$

$$p_{k'}{}^* = \frac{\left((\beta+\gamma)^2 - \gamma^2\right)(\alpha_{k'} - Q_0) + \gamma(\beta+\gamma)(\alpha_k - Q_0) + \gamma^2(\alpha_{k'} - Q_0)}{(\beta+\gamma)\left((\beta+\gamma)^2 - \gamma^2\right)}, k,k' = A,B\ \ ,k' \neq k \tag{3-13}$$

$$\pi_k(c_{jk}, c_{ik'}) = \left[\alpha_k - \beta p^*{}_k + \gamma\left(p_{k'}^o - p^*{}_k\right)\right]\left[p^*{}_k - c_k - \left(\mu_k c_{jk}/2\right)\right] - A_k \tag{3-14}$$

$\forall i,j = 1,2,\ \ k,k' = A,B\ \ ,k' \neq k$

Finally, set these obtained prices in $\pi_k(c_{jk}, c_{ik'})$ introduced previously. Similarly, compute $\pi_{k'}(c_{jk}, c_{ik'})$. For this game with simultaneous moves, the mixed strategies contain y and x.

$$\text{y} = \text{p}(\text{retailer A beleives that retailer B rejects the offered discount}) \tag{3-15}$$

$$\text{x} = \text{p}(\text{retailer B beleives that retailer A rejects the offered discount}) \tag{3-16}$$

Therefore, the expected utility for each strategy profile can be computed as (3-17) and (3-18).

$$E(u_A(c_{1A}, y)) = \int_{\mu_1}^{\mu_2}\left(y \times \pi_A(c_{1A}, c_{1B}) + (1-y) \times \pi_A(c_{1A}, c_{2B})\right) f(\mu_B) d\mu_B \tag{3-17}$$

$$E(u_A(c_{2A},y))=\int_{\mu_1}^{\mu_2}\left(y\times\pi_A(c_{2A},c_{1B})+(1-y)\times\pi_A(c_{2A},c_{2B})\right)f(\mu_B)d\mu_B \tag{3-18}$$

Actually, (3-17) and (3-18) represent the expected profit of retailer A when he/she decides to reject and accept the offered discount, respectively.

By setting these two relations equal to each other, the mixed strategy for the two retailers are obtained as displayed in (3-19) and (3-20).

$$y\times\left(\pi_A(c_{1A},c_{1B})-\pi_A(c_{2A},c_{1B})-\pi_A(c_{1A},c_{2B})+\pi_A(c_{2A},c_{2B})\right)+\pi_A(c_{1A},c_{2B})-\pi_A(c_{2A},c_{2B})=0$$

$$=\frac{\pi_A(c_{2A},c_{2B})-\pi_A(c_{1A},c_{2B})}{\left(\pi_A(c_{1A},c_{1B})-\pi_A(c_{2A},c_{1B})-\pi_A(c_{1A},c_{2B})+\pi_A(c_{2A},c_{2B})\right)} \tag{3-19}$$

$$=\frac{\pi_B(c_{2A},c_{2B})-\pi_B(c_{2A},c_{1B})}{\left(\pi_B(c_{1A},c_{1B})-\pi_B(c_{2A},c_{1B})-\pi_B(c_{1A},c_{2B})+\pi_B(c_{2A},c_{2B})\right)} \tag{3-20}$$

We know that probabilities are equal or more than 0. Therefore, if we have $1\geq y\geq 0$ and $1\geq x\geq 0$, then Nash equilibrium could be computed as follows. Let

$$\Delta_1=\left(\pi_A(c_{2A},c_{2B})-\pi_A(c_{1A},c_{2B})\right)/\left(\pi_A(c_{1A},c_{1B})-\pi_A(c_{2A},c_{1B})-\pi_A(c_{1A},c_{2B})+\pi_A(c_{2A},c_{2B})\right) \tag{3-21}$$

$$\Delta_2=\left(\pi_B(c_{2A},c_{2B})-\pi_B(c_{2A},c_{1B})\right)/\left(\pi_B(c_{1A},c_{1B})-\pi_B(c_{2A},c_{1B})-\pi_B(c_{1A},c_{2B})+\pi_B(c_{2A},c_{2B})\right) \tag{3-22}$$

Nash Bayesian equilibrium for this game consists of

$$\begin{cases} \textit{retailer A rejects the offered discount} \overset{\text{if}}{\rightarrow} y>\Delta_1 \\ \textit{retailer A accepts the offered discount} \overset{\text{if}}{\rightarrow} \mathrm{y}\leq\Delta_1 \end{cases} \tag{3-23}$$

$$\begin{cases} \textit{retailer B rejects the offered discount} \overset{\text{if}}{\rightarrow} \mathrm{x}>\Delta_2 \\ \textit{retailer B accepts the offered discount} \overset{\text{if}}{\rightarrow} \mathrm{x}\leq\Delta_2 \end{cases} \tag{3-24}$$

Lemma 2: In a competitive situation with an all-unit quantity discount offered by a monopolistic supplier, if all optimum prices for both retailers set in their domains, then rejecting the offered discount is the strictly dominant strategy of retailer k. In other words, we have for retailer k:

$$\pi_k(c_{2k},c_{ik'})\leq\pi_k(c_{1k},c_{ik'}),\quad \forall i=1,2,\ k,k'=A,B\ ,k'\neq k \tag{3-25}$$

Proof (lemma 2): We have

$$\pi_k(c_{2k},c_{ik'})-\pi_k(c_{1k},c_{ik'})=$$

$$\frac{(\beta+\gamma)^2(2+\mu_k)(c_{2k}-c_{1k})}{B'}\left\{\alpha_k+\frac{(\beta+\gamma)^3(2+\mu_k)(c_{2k}+c_{1k})+(\beta+\gamma)\gamma\left(A'_1+A'_2\right)}{B'}\right\} \tag{3-26}$$

where $B'=4(\beta+\gamma)^2-\gamma^2$, $A'_1=2\alpha_{k'}+\frac{\gamma\alpha_k}{\beta+\gamma}$, $A'_2=\frac{(\beta+\gamma)(4+\mu_{k'}+\mu_k)c_{ik'}}{2}$. Since $c_{2k}<c_{1k}$, then rejecting the offered discount is the strictly dominant strategy of each retailer. It means:

$$\pi_k(c_{2k},c_{ik'})\leq\pi_k(c_{1k},c_{ik'}),\forall i=1,2,\ k,k'=A,B\ ,k'\neq k \tag{3-27}$$

For example, if all prices set in their related domains, we have from relations (3-14) and (3-19),

$$y=\left\{2B\alpha_k+\left\{2(\beta+\gamma)^3(2+\mu_k)(c_{2k}+c_{1k})+(\beta+\gamma)\gamma\left(2A'_1+(\beta+\gamma)\left(4+\mu_k+\mu_{k'}\right)c_{2k}\right)\right\}\right\}/$$
$$\{\gamma(\beta+\gamma)^3\left(4+\mu_k+\mu_{k'}\right)(c_{2k}-c_{1k})\},\ k,k'=A,B\ ,k'\neq k \tag{3-28}$$

Since $c_{2k}<c_{1k}$, then $y<0$. One of the requirements of the probability distribution is that all probabilities are equal or more than 0. Therefore, because we have $y<0$, then rejecting the offered discount is Nash equilibrium for retailer k. In addition, it is true for the other retailer.

But in other cases, in which all optimum prices for both retailers do not set in their domain, investigating Nash equilibrium for each retailer is more complicated. In these situations, it is necessary to use Eq. (3-19) and (3-20) to obtain the best response of each retailer to its rival.

5. Discussion and Computational Results

5. 1 Numerical Example

In this section, the distribution of the produced fertilizer from an Iranian special brand which is in restriction of two retailers, denoted by A and B, is considered to show how the proposed algorithm works. In this case, we have the following inputs:

$A_A = 100\$$ $\quad \mu_A = 24\%$ $\quad \alpha_A = 700\ unit/year$ $\quad \beta = 5\ unit/\$,$

$A_B = 120\$$ $\quad \mu_B = 18\%$ $\quad \alpha_B = 650\ unit/year$ $\quad \gamma = 2\ unit/\$$

The holding cost of retailers A and B are their private information, but $\mu_k, (k = \text{A}, \text{B})$ is assumed to be uniformly distributed, i.e., $\mu_k \sim U[0.15, 0.25]$ and it is their common knowledge.

The following all-unit quantity discount scheme is offered by the monopolistic manufacturer:

$$\begin{cases} \$30, & 0 < q_k \leq 350 \\ \$27, & 350 < q_k \end{cases}$$

Two cases may happen:

1- The retailers tend to share their information prior to the game being played,

2- Holding cost rate of each retailer is its private information.

It is desirable to investigate if sharing information between the retailers is profitable for each retailer or not.

Case (1): discount game with complete information:

Based on the proposed algorithm, we have

Step 1: The optimal and threshold prices for each case are computed based on (2-7) and (2-8) and displayed in Tables 2 and 3, respectively.

Table 2. The Optimal Prices

Retailer B / Retailer A	**Rejecting the discount (buying at a price of 30)**	**Accepting the discount (buying at a price of 27)**
Rejecting the discount (buying at a price of 30)	(77.346,73.83)	(76.025,64.578)
Accepting the discount (buying at a price of 27)	~~(**70.827,72.89**)~~	(67.77,62.22)
	~~(**68.55**,63.16)~~	
	(68.04,63.16)	

Table 3. The Computed Values for $\boldsymbol{p_k^o}$

Retailer B / Retailer A	**Rejecting the discount (buying at a price of 30)**	**Accepting the discount (buying at a price of 27)**
Rejecting the discount (buying at a price of 30)	(71.09,64.95)	(68.45,64.578)
Accepting the discount (buying at a price of 27)	~~(68.55,63.16)~~	(67.77,62.22)
	~~(68.04,62.44)~~	
	(68.04,62.29)	

Comparing the two matrices, the main obtained result is that some bold prices do not set in their related span, and conditions of lemma 1 have not been met. These comparisons are displayed in Figure 5. As seen in this figure, only for the strategy (27\$,30\$), p_A^* does not set in its acceptable span. Therefore, for three other strategies, the best prices are accepted and for the strategy (27\$,30\$), we must set $p_A^* = p_A^o$ and compute p_B^*, p_A^o and p_B^o again. We investigate if p_A^* and p_B^* set in their acceptable spans or not. The results are displayed in Tables 2 and 3 for the best prices and the threshold prices for the strategy (27\$,30\$). As seen, this process continues until both prices set in their related domains.

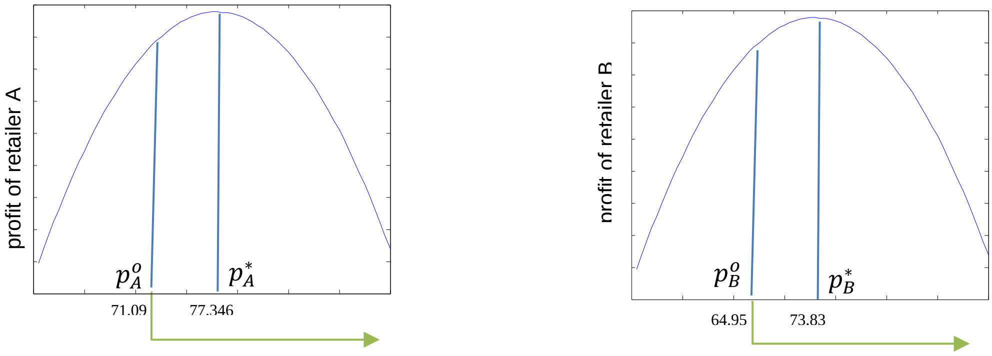

Figure 5.a. The Acceptable span for Prices of Two Retailers for (30$,30$)

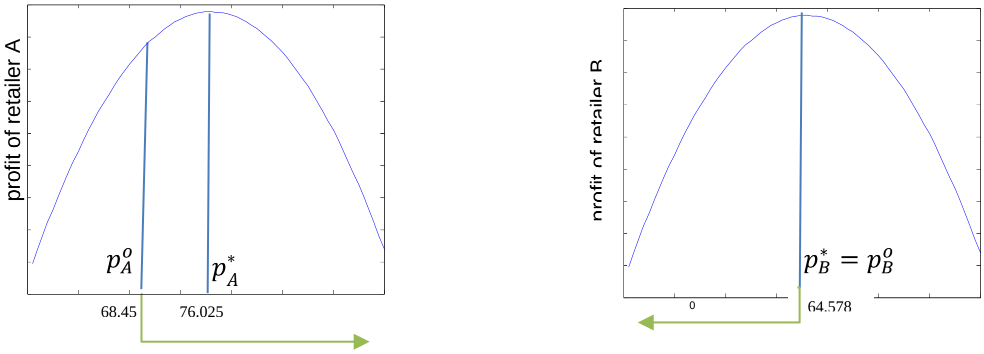

Figure 5.b. The Acceptable Span for Prices of Two Retailers for (30$,27$)

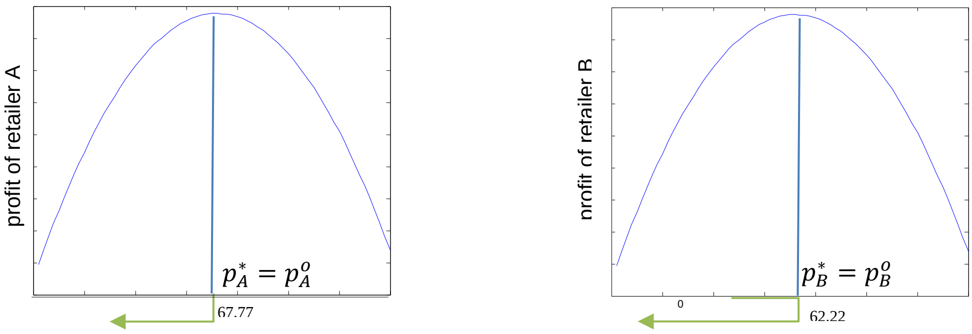

Figure 5.c. The Acceptable Span for Prices of Two Retailers for (27$,27$)

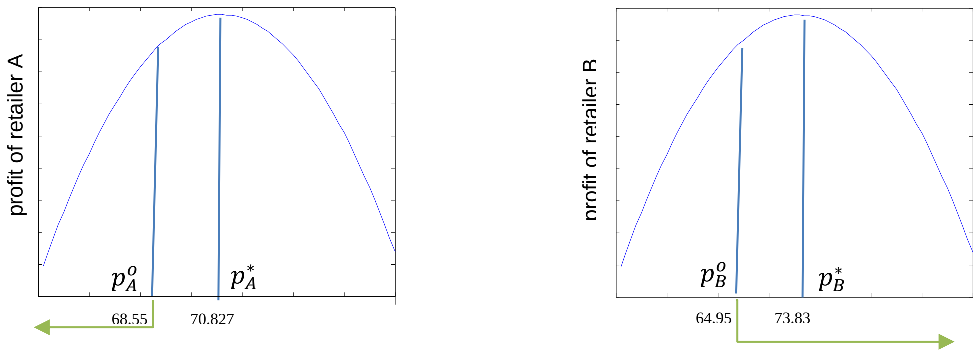

Figure 5.d. The Acceptable Span for the Prices of Two Retailers for (27$,30$)

Figure 5. The acceptable Span for the Prices of Two Retailers for Their Different Strategies

Step 2: Given these optimal prices for each case, the pay-off matrix is computed based on relations (2-20) and (2-21), as displayed in Table 4.

Table 4. The Pay-Off Matrix for Numerical Instance with Complete Information

Retailer A \ Retailer B	**Rejecting the discount (buying at a price of 30)**	**Accepting the discount (buying at a price of 27)**
Rejecting the discount (buying at a price of 30)	(13297,11721)	(12499,12181)
Accepting the discount (buying at a price of 27)	(13431,10477)	(13037,11356)

Step 3: obtain Nash equilibrium for this game. As seen, accepting the offered discount is the strictly dominant strategy for two retailers.

Case (2): discount game with incomplete information:

The pay-off matrix is as displayed in Table 5.

Table 5. The Pay-Off Matrix for Retailers with Incomplete Information

Retailer A \ Retailer B	**Rejecting the discount (buying at a price of 30)**	**Accepting the discount (buying at a price of 27)**
Rejecting the discount (buying at a price of 30)	(13309.8,11695.43)	(12499.5,12150.8)
Accepting the discount (buying at a price of 27)	(13037.7,11412)	(13038.6,11356.8)

Because there is no pure Nash equilibrium, it is necessary to compute mixed strategy for each retailer, and Nash Bayesian equilibrium for this game is as follows:

$$\begin{cases} \textit{retailer A rejects the offered discount} \xrightarrow{\text{if}} y > 0.664 \\ \textit{retailer A accepts the offered discount} \xrightarrow{\text{if}} \mathrm{y} \le 0.664 \end{cases}$$

$$\begin{cases} \textit{retailer B rejects the offered discount} \xrightarrow{\text{if}} \mathrm{x} > 0.108 \\ \textit{retailer B accepts the offered discount} \xrightarrow{\text{if}} \mathrm{x} \le 0.108 \end{cases}$$

By comparing the results obtained from the two mentioned cases, for ($\mathrm{y} \le 0.664$, $\mathrm{x} \le 0.108$) the profit of each retailer is not influenced by having or not having information about the cost structure of its competitor and two retailers can obtain the same profit in two cases. In other situations (other than ($\mathrm{y} \le 0.664$, $\mathrm{x} \le 0.108$)), discount program may fail. Considering the benefits of sharing information, the manufacturer can persuade retailers to negotiate about sharing their information.

5. 2 Sensitivity Analysis

A systematic sensitivity analysis is performed to investigate which parameters have the highest information value for the retailers, and on which parameters they should spend more time and money to accurately estimate or negotiate for sharing it. To this end, the value of each given parameter ($A_k, \mu_k, c_{1k}, c_{2k}$, Q_0, α_k, β and γ) is changed in this instance for the case with complete information, and the effect on Nash equilibrium is reported.

Table 6. Sensitivity Analysis for the Cost Parameters

Original values	New values for retailer k'	New values for retailer k	p^*_k	$p^*_{k'}$	$\pi_k(c_{2k}, c_{2k'})$	$\pi_{k'}(c_{2k}, c_{2k'})$	Selected strategy
$A_k = 100$, $A_{k'} = 120$	$A_{k'} = 100$	$A_k = 80$	67.78	62.2	13058	11377	(yes,yes)
		$A_k = 120$	67.78	62.2	13018	11377	(yes,yes)
	$A_{k'} = 140$	$A_k = 80$	67.78	62.2	13058	11337	(yes,yes)
		$A_k = 120$	67.78	62.2	13018	11337	(yes,yes)
$\mu_k = 24\%$, $\mu_{k'} = 18\%$	$\mu_{k'} = 16\%$	$\mu_k = 22\%$	67.78	62.22	13133	11452	(yes,yes)
		$\mu_k = 26\%$	67.78	62.22	12944	11452	(yes,yes)
	$\mu_{k'} = 20\%$	$\mu_k = 22\%$	67.78	62.22	13133	11263	(yes,yes)
		$\mu_k = 26\%$	67.78	62.22	12944	11263	(yes,yes)
$c_{1k} = 30$, $c_{2k} = 27$	$c_{1k} = 27$, $c_{2k} = 24$		67.8	62.22	14214	12502	(yes,yes)
	$c_{1k} = 33$, $c_{2k} = 30$		71.03	73.6	13002	10299	(yes,no)
$Q_0 = 350$	300		71.77	68.79	13698	12278	(yes,yes)
	400		62.92	67.79	12973	10282	(yes,no)

Table 7. Sensitivity Analysis for the Demand Parameters

Original values	New values for retailer k'	New values for retailer k	p^*_k	$p^*_{k'}$	D_k	$D_{k'}$	$\pi_k(c_{2k},c_{2k'})$	$\pi_{k'}(c_{2k}, c_{2k'})$	Selected strategy
$\alpha_k = 700$, $\alpha_{k'} = 650$	$\alpha_{k'} = 600$	$\alpha_k = 650$	62	67.14	350.28	253.8	11030	8631	(yes,no)
		$\alpha_k = 750$	78.7	70.45	456.6	264.3	16377	9859	(yes,no)
	$\alpha_{k'} = 700$	$\alpha_k = 650$	62.2	67.78	350.02	349.9	11094	13302	(yes,yes)
		$\alpha_k = 750$	77.7	72.22	349.98	350.0	16538	14857	(yes,yes)
$\beta = 5$	3		107	103.0	368.32	35.03	28372	25635	(yes,yes)
	7		51.6	57.62	350.03	234.8	7409	5731	(yes,no)
$\gamma = 2$	1		70.9	75.7	350	266.7	14152	11351	(yes,no)
	3		67.2	62.72	350	350.0	12861	11534	(yes,yes)

The following points and managerial implications can be inferred from the analyses of the results shown in Tables 6 and 7:

(1) As either the prices paid per purchased unit (i.e., c_{1k}, c_{2k}) or market base for retailer k (i.e., α_k) increases for each retailer (e.g., retailer k), p^*_k rises. In addition, p^*_kreduces with higher amount of Q_0. If the manufacturer is working in a competitive market, he must control his costs to decrease the wholesale prices for both the cases with or without discount (i.e., c_{1k}, c_{2k}) and increase the break point of discount (i.e., Q_0). This leads to lower retailing prices, and due to absorbing more customers, it can lead to maintaining or increasing retailer's market share.

(2) For maximizing profit, the companies should give more attention to increasing the demand than reducing the costs. As displayed in Table 7, among demand parameters, it could be observed that primarily β and secondarily α are the most influential factors on the demand and profit functions. Applying these cost controlling strategies by the manufacturer is more vital for the products with more demand elasticity. If a factor besides price changes, such as a change in consumers' preferences or increase in consumers' income, more products will be demanded even if the price remains the same.

(3) Among the cost parameters ($c_{1k}, c_{2k}, A_k, \mu_k$ and Q_0), the most important parameter for profitability is the purchase costs with and without discount, c_{1k} and c_{2k}, followed by the break point of discount Q_0. The main managerial point of these results is the detection of the most important parameters for retailers. It means that if the retailers spend more time and money to accurately estimate or negotiate for sharing these parameters, they could increase the validity of their decisions.

(3) It is worth mentioning that the retailers' profits do not increase always with an increase in the selling price. As seen, for different values of ordering cost and holding cost rate of retailer k, (i.e., A_k and μ_k), while the retailer prices remain fixed, a higher profit can be obtained for this retailer. Also for γ, a higher profit can be obtained for retailer k' with a lower unit price $p^*_{k'}$.

(4) More importantly, the selected strategy by each retailer could change based on their parameters. In this direction, as seen in Tables 6 and 7, two retailers accept discounts and their decisions remain unchanged with changes in A_k, and μ_k. For other parameters, it could be seen that a change in the

value of parameters results in changing the discount strategy selected by the retailer. Therefore, if each retailer makes mistake in estimating the parameters of the other retailer, he may take the wrong decision whether to adopt a regular or special-order policy and a wrong strategy in ordering the products. In addition, it could be seen that all parameters do not have the same importance in estimation.

5. 3 Benefits and Risks of Considering Incomplete Information

To investigate the benefits of considering incomplete information assumption in the model, our results must be compared with the case where there is no lack of information. In these cases, two retailers have the same perceptions of all problem parameters and both players decide based on the amounts of μ_k and $\mu_{k'}$. To evaluate these two cases and compare them with each other, we used I and R as two well-known evaluation criteria that are defined as follows (Bayrak and Bailey, 2008):

I% represents the percent change in the profit of retailer k when incomplete information is considered and is computed as follows.

$$I\% = \left| \frac{\pi_{k,2} - \pi_{k,1}}{\pi_{k,1}} \right| \times 100$$

$\pi_{k,1}$: The true profit of retailer k when he/she assumes that there is no incomplete information,

$\pi_{k,2}$: The best obtained profit of retailer k with incomplete information.

Then, this relation is described in more details by using the previous numerical example.

First case: assume that retailer k does not consider the incomplete information assumption on μ_k and $\mu_{k'}$. Then for the previous problem, she/he expects to obtain 13037.4 $ for selecting strategy (yes, yes) by the two retailers, i.e., a Nash equilibrium for complete information cases. But in this case, because retailer k' takes the incomplete information assumption into consideration, (yes, yes) may not be a Nash equilibrium.

If $y \geq 0.664$, then retailer k' assumes that retailer k rejects the offered discount and based on this assumption he/she accepts the offered discount. However, as described, retailer k accepts the discount. By these descriptions, strategy (yes, yes) is selected by two retailers and retailer k could obtain 13038$. Then we can set $\pi_{k,1} = 13038$.

Similarly, if $y < 0.664$, then (yes, no) is selected by retailers, we can set $\pi_{k,1} = 13037.7$.

Second case: assume that two retailers consider the incomplete information assumption on μ_k and $\mu_{k'}$. Then, for example we can set $\pi_{k,2} = 12499.5$ for ($y > 0.664$, $x \leq 0.108$) and $\pi_{k,2} = 13309.8$ for ($y > 0.664$, $x > 0.108$), and so on.

To compare these cases, I is computed for different values of (y, x), as displayed in Table 8.

Table 8. Calculating Benefit of Considering Incomplete Information Assumption

	$\pi_{k,1}$	$\pi_{k,2}$	I (%)
($y > 0.664$, $x \leq 0.108$)	13038	12499.5	6.2
($y > 0.664$, $x > 0.108$)	13038	13309.8	2.08
($y \leq 0.664$, $x \leq 0.108$)	13037.7	13038.6	0
($y \leq 0.664$, $x > 0.108$)	13037.7	13037.7	0

In the case under consideration with incomplete information, it is assumed that each retailer knows the parameters of the other retailer's distribution. However, if this is not the case, then some profitable pricing strategies may be left more vulnerable. Consequently, if retailer k assumes incorrect distribution parameters for retailer k' (i.e., μ_1, μ_2 in $\mu_{k'} \sim U[\mu_1, \mu_2]$), this lack of information results in a risk represented with R% and is calculated as follows.

$$R\% = \left| \frac{\pi_{k,4} - \pi_{k,3}}{\pi_{k,4}} \right| \times 100$$

$\pi_{k,3}$: The profit of retailer k when the estimates of this retailer for distribution parameters of retailer k' are not correct.

$\pi_{k,4}$: The profit of retailer k when his/her estimates for distribution parameters of retailer k' are accurate.

It is assumed that retailer k does not have an accurate estimation of the distribution parameter of retailer k'. To compute this value, it is necessary to solve the problem two times (Q is defined as the accuracy rate for estimation of μ_1 and μ_2).

First (for $Q < 1$): Retailer k finds his/her Nash price based on his/her inaccurate estimations of μ_1 and μ_2. Then his/her profit is obtained based on the accurate values of μ_1 and μ_2. This profit of retailer k is known as $\pi_{k,3}$.

Second (for $Q = 1$): Now it is assumed that the accurate estimation of distribution parameters is on hand. In this case, the profit of retailer k is $\pi_{k,4}$, which is obtained as the best possible obtained objective function value of the considered game.

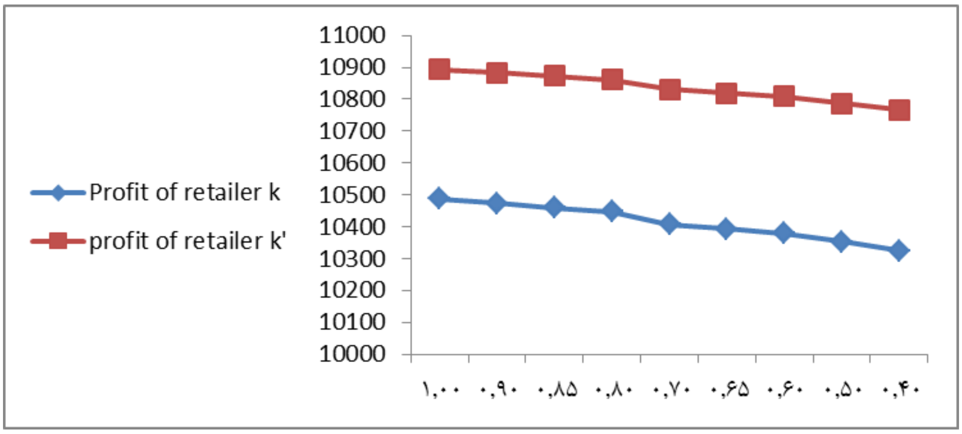

Figure 6. The risks Over Different Values of Estimation Accuracy for Strategies (No, No)

Displaying R% over different values for Q, it is observed in Figure 6 that the incomplete information model can formulate the real situation in a better manner as retailer k estimates the parameters of retailer k' with less accuracy (that is, when Q decreases). The main managerial points inferred from this analysis is that the retailers need to spend more budget to obtain more accurate knowledge about their rivals.

6. Conclusions and Future Studies

Although considering the behavior of other retailers in decision making may complicate the discount problem, it helps retailers decide correctly and exactly in a competitive market. Different from the conventional literature on supply chain coordination, we mainly focus on situations where there are two competing retailers that want to decide about their optimal prices under an offered discount by a monopolistic supplier. With both assumptions that holding cost rate of each retailer is common knowledge and private information, we modeled these situations as a strategic form game with complete and incomplete information, respectively, and found Nash equilibriums for these cases. We have shown in Lemmas (1) and (2) that if both retailers have complete or incomplete information about their rival's cost structure, under some special conditions, rejecting the offered discount is the strictly dominant strategy of retailer. However, for a Cournot competition, Navidi and Bidgoli (2011) approved this for a special case in a game with complete information in which all optimum ordered quantities for every strategy profile were set in their rejection domains, accepting discount for every retailer as their strictly dominant strategy. However, in incomplete information case, they could not present any clear result.

The sensitivity analysis showed that the retailers' profits are mostly influenced by the demand parameters. This means that for maximizing the profit of retailers, the companies should give more attention to increasing the demand rather than reducing the costs. The retailers need to first direct their efforts towards absorbing more customers. The second way to boost profitability of retailers is to reduce their costs, especially the purchasing costs with and without discount and the break point of discount via negotiations with the manufacturer. Interestingly, the third way to increase the profit of retailers is to decrease the retailing prices.

There are still many questions that need to be studied. In this paper, for simplicity, we assumed the pricing schedule has only one breakpoint. Multiple price break structure case should be examined in the future works. The model can be extended to the case in which there are more competing retailers than two. The case of nonlinear demand functions and the case in which the market demands are the functions of both retail prices and services are also very interesting.

References

Alfares, H. K., & Ghaithan, A. M. (2016). Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. *Computers & Industrial Engineering*, 94, 170-177.

Bayrak, H., & Bailey, M. D. (2008). Shortest path network interdiction with asymmetric information. *Networks*, *52*(3), 133-140.

Benton, W. C., & Park, S. (1996). A classification of literature on determining the lot size under quantity discounts. *European Journal of Operational Research*, *92*(2), 219-238.

Forghani, K., Mirzazadeh, A., & Rafiee, M. (2013). A price-dependent demand model in the single period inventory system with price adjustment. *Journal of Industrial Engineering*, 2013.

Li, X., Nukala, S., & Mohebbi, S. (2013). Game theory methodology for optimizing retailers' pricing and shelf-space allocation decisions on competing substitutable products. *The International Journal of Advanced Manufacturing Technology*, *68*(1), 375-389.

Mahmoodi, A. (2019). Joint pricing and inventory control of duopoly retailers with deteriorating items and linear demand. *Computers & Industrial Engineering*, *132*, 36-46.

Mahmoodi, A. (2020). Stackelberg–Nash equilibrium of pricing and inventory decisions in duopoly supply chains using a nested evolutionary algorithm. *Applied Soft Computing*, *86*, 105922.

Melnikov, S. (2017). Bertrand-Nash Equilibrium in the Retail Duopoly Model under asymmetric costs. *International Journal of Engineering*, *30*(6), 859-866.

Navidi, H., & Bidgoli, M. M. (2011). An all-unit quantity discount model under a Cournot competition with incomplete information. *International Journal of Management Science and Engineering Management, 6*(5), 393-400.

Saha, S., Nielsen, I. E., & Moon, I. (2021). Strategic inventory and pricing decision for substitutable products. *Computers & Industrial Engineering*, 160, 107570.

Shah, N. H. (2014). Ordering policy for inventory management when demand is stock-dependent and a temporary price discount is linked to order quantity. *Investigación Operacional*, *33*(3), 233-244.

Shi, J., Zhang, G., & Lai, K. K. (2012). Optimal ordering and pricing policy with supplier quantity discounts and price-dependent stochastic demand. *Optimization*, *61*(2), 151-162.

Xiao, T., & Qi, X. (2008). Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers. *Omega*, *36*(5), 741-753.

Zhang, J. L., Lee, C. Y., & Chen, J. (2009). Inventory control problem with freight cost and stochastic demand. *Operations Research Letters, 37*(6), 443-446.