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Regime changes between Bitcoin and six other assets using Copula model with Markov switching

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ABSTRACT

Examining the structure of dependence between financial assets and the effects of their Co-movement is one of the important issues in financial markets. The corresponding copula is one of the most computationally convenient ways to describe the dependency structure. This paper examines regime change probability and the best copula model between Bitcoin and six other assets from 2018 to 2021. First, using the ARMA-GARCH model, the marginal distribution functions for all assets and residuals are calculated. Then, by using the obtained residuals, 11 models of copula and six models of combined Copula with Markov switching were implemented. The model that has the best function for constructing combined distribution functions is selected. Finally, the regime probabilities each time are calculated from the best-fitted model. The results show that in the study period, for Bitcoin-Ethereum, Bitcoin-Cardano, and Bitcoin-Gold pairs MS-CT, for Bitcoin-Binance coin and Bitcoin-Ripple pairs MS-CRG and MS-CN for Bitcoin-Oil pair have the best performance. Furthermore, the probabilities of regime change between each asset at each time were calculated and described.

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1. Introduction

In the last decade, cryptocurrencies have become a $3 trillion market that can be seen to affect other markets. So many global investors, estimated at 300 million, have invested in these currencies, which is growing over time. By these definitions, it is understood that according to the expected effects of money movements and investors' expectations, it can be expected that the impact of this market on other financial markets will be more significant in the future. The financialization of Bitcoin and its characteristics of high returns and high volatility mean that any evidence of risk spillovers between Bitcoin and other financial assets would have a major impact on risk management, investment portfolio management, and policy-making (Baur, Hong & Lee, 2018) and (Gon & Lin, 2021). Specifically, Bitcoin shares some similarities with Gold and USD. Besides, several famous financial media outlets, such as CNN, have labeled Bitcoin as New Gold, and CFTC has also declared virtual money a commodity like Gold. This means that if users of Bitcoin are rational, Bitcoin has some intrinsic value like Gold. It has also proved that Gold maximizes the attribute of value storage by sacrificing liquidity while USD maximizes liquidity by sacrificing storage value, but Bitcoin combines the characteristics of Gold and USD and can be used for payments, portfolio management, and risk hedging like Gold and USD (Dyhrberg, 2016). Other studies have also attempted to explore Bitcoin's ability to hedge market risks and found that it can be used for payment, portfolio management, and risk hedging (Bouri et al, 2020; Das et al, 2020; Klein et al, 2018; Kliber et al, 2019 Shahzad et al, 2019).

Bitcoin and Gold relationship, as well as Bitcoin and USDs, has also been explored in recent years (Klein et al., 2018; Baur et al., 2018; Wang et al., 2019; Bai et al., 2021).

So, it is vital to look at the relationship between Bitcoin and other financial assets at any given time. This allows us to understand the co-movement of financial assets with each other and to extract the appropriate decision in trading positions. Therefore, the objective of this research is to find better and more efficient models than copula models to evaluate the dependence structure and simultaneously the regime switching between assets in the upward and downward trends.

Therefore, this paper uses an ARMA-GARCH-t process to create the marginal distribution functions for each time series residual and uses the Markov switching copula approach to find the best-fitted copula model and calculate the probabilities of presence in each regime.

As seen, examining the correlation structure and choosing the appropriate hypothesis for this structure is variable in different studies, and therefore, it has changed the results of different articles. The Novelties of this paper are as follows:

1. Combining the copula method with the Markov switching model.
2. Using t-Student marginal distribution functions in the copula method.
3. Building models of dependence structure between Bitcoin, other cryptocurrencies and gold and oil using Markov switching copula

The article is organized as follows. The literature review is presented in section 2, Research methodology is presented in Section 3, Empirical results are presented in Section 4 and the article is concluded in Section 5.

2. Literature Review

Research studies on the relationship between Bitcoin and Gold, as well as between Bitcoin and USD, yield some controversial results. For example, Baur et al. (Baur et al., 2018) found that Bitcoin returns are uncorrelated to Gold and USD returns by linear regression analysis, whereas Bouri et al. (Bouri et al. 2018) demonstrated that Bitcoin returns correlated closely with Gold and USD returns by a smooth transition VAR-GARCH-in-mean model. (Wang et al, 2019) proved that there is no return spillover between Bitcoin and Gold/currency, and volatility spillover exists only between Bitcoin and Gold by a VAR-GARCH-BEKK model. Interestingly, using different extensions to the NARDL model, (Bouri et al. 2018) found a significant negative influence of Gold and USD prices on Bitcoin prices, whereas (Jareño et al. 2020) showed that the short-term and long-term influence of Gold on Bitcoin is asymmetric. These opposite results may be because the correlation between Bitcoin and Gold/USD is dynamic.

Some studies have examined the effect of interdependence between financial assets and other approaches (Awartani & Maghyereh, 2013) that employ a DCC-GARCH model and show the correlation between stock market returns and Oil price changes varies over time. Although the DCC-GARCH model allows for the time-varying conditional correlation, it fails to reproduce the non-linear dependence that may exist between the variables and does not provide information about the tail dependence. The tail dependence corresponds to the possibility of joint events such as low or high extreme events. To do so, an alternative approach based on copula functions has been adopted. The main advantage of the copulas lies in separating the dependence structure from the marginals without making any assumptions about the distribution. Using several copula functions, (Nguyen & Bhatti, 2012) provide evidence of left tail dependence in Vietnam, whereas there is no tail dependence in China. In the case of six CEE countries (Bulgaria, Czech Republic, Hungary, Poland, Romania, and Slovenia), left tail dependence was also found (Aloui et al., 2013).

Prior studies like (Wang et al., 2013) apply Markov-switching copula functions to examine the dependence between international stock markets. These studies consider a finite mixture of conditional bivariate copulas, where the copula parameter is fixed, but the functional form of the copula functions follows a Markov-switching model. This approach depends on the selection of suitable copulas.

This paper proposes an approach in which the copula function remains constant, but the copula parameter is subject to change over time according to a Markov-switching model (see Ji et al, 2020) for an Application to stock market risk spillover from the US to other G7 countries).

Recently, researches such as (Rehman and Tiwari, 2023) investigate the dependence structure among the seven emerging stock markets by employing a dependence-switching copula model.

Also (Niu et al , 2023) propose a Markov-switching mixed-Clayton (Ms-M-Clayton) copula model that combines a state transition mechanism with a weighted mixed-Clayton copula for investigate the dynamic risk dependence between Chinese and mature stock markets in the Americas, Europe, and Asia–Oceania regions.

Some studies, like (Fülle and Herwartz, 2022), used copula with Markov, like (Fülle and Herwartz, 2022), used copula with Markov switching for portfolio forecasting. They apply a Markov switching copula multivariate GARCH (MS-C-MGARCH) model for this purpose. Before that, (Fülle and Herwartz, 2022) suggested a new Markov switching approach to multivariate volatility modeling for improving the dynamic assessment of financial market interdependencies and tried to answer this question: Is Gold Always a Safe-Haven?

From a financial science point of view, investing in different assets has always been an essential matter. Moreover, the question that investors faced was, which asset should we invest in? Paying attention to return has always been considered in the topic of financial research. Over time, in 1952, with the model that (Markowitz, 1952) developed, the majority of the articles in this field were placed on investment and building an assets portfolio by considering the two issues of risk and return. Recently, beyond the Markowitz model (mean-variance model), ESG factors (environmental, social, and Governance) and economic/business cycles have also been taken into consideration. This study focused on the modeling of economic/business cycles and their regime switching via the dependence structure modeling between assets.

3. Methodology

3.1. Model for Marginal Distribution

Before estimating the copula function, we employ the ARMA(1,1)-GARCH(1,1)-t model to construct the marginal distribution and calculate residuals of Bitcoin, Ethereum, Cardano, Binance coin, Ripple, Oil, and Gold. In this model, the conditional mean is calculated by an ARMA (1,1) model:

$$r_t = \varphi_0 + \sum_{i=1}^{m} \varphi_j r_{t-i} + \varepsilon_0 + \sum_{j=1}^{n} \psi_j \varepsilon_{t-j} = \mu_t + \varepsilon_t$$

In which φ_0 and φ_j are the parameters to be estimated, m is the lag order of returns ,n is the order of the moving average, and $\varepsilon_t = \sigma_t z_t$, $z_t \sim$ i.i.d. D(0, 1) is the error item. The conditional variance, σ^2_t, is given by a GARCH model:

$$\varepsilon_t = \sigma_t Z_t \ , \quad Z_t \ i.i.d \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2$$

where α_0, α_j, and β_j are the parameters to be estimated and sum (α_j)+sum(β_j) should be smaller than 1, p and q are the lag order of the ARCH term and GARCH term, respectively. (Brooks, 2002) proves that using a GARCH-class model with one lag order is sufficient to describe the volatility clustering in asset returns. So, we set m, n, p, and q to be one, constructing an ARMA (1,1)-GARCH (1,1) model. To capture the fat tail and asymmetry distribution features of Bitcoin, Ethereum, Cardano, Binance coin, Ripple, Oil, and Gold returns, we assume the i.i.d. random variable, z_t, follows the t-distribution (Nguyen & Bhatti, 2012).

3.2. Copula Function

We explore the dependence structure between Bitcoin, Ethereum, Cardano, Binance coin, Ripple, Oil, and Gold using some bivariate copula functions. According to Sklar (Sklar, 1959), a multivariate copula can couple multi-marginal distributions to represent a joint distribution function as:

$$F\left(x_1,\ldots,x_d\right)=C\left(F_1\left(x_1\right),\ldots,F_d\left(x_d\right)\right)$$

For two marginal distributions:

$$F\left(x_1,\ x_2\right)=C\left(F_1\left(x_1\right),F_2\left(x_2\right)\right)$$

where F(x, y) is the joint distribution function of two random variables; $F_1(x_1)$ and $F_2(x_2)$ are the marginal distribution functions of X_1 and X_2, respectively; and C(u, v) for $u = F_1(x_1)$ and $v= F_2(x_2)$.

Sklar's theorem shows that when the variables are continuous, each multivariate probability distribution function can be represented using a marginal distribution and a dependent structure, which is inferred as follows:

$$\mathrm{f}\left(\mathrm{x}_1,\ \ldots,\ \mathrm{x}_\mathrm{n}\right)=\frac{\partial^\mathrm{n}\mathrm{C}\left(\mathrm{F}_1\left(\mathrm{X}_1\right),\ \ldots,\ \mathrm{F}_\mathrm{n}\left(X_\mathrm{n}\right)\right)}{\partial\mathrm{F}_1\left(\mathrm{X}_1\right)\ldots\partial\mathrm{F}_\mathrm{n}\left(\mathrm{X}_\mathrm{n}\right)}\times\prod_{\mathrm{j}=1}^{\mathrm{n}}\frac{\partial\mathrm{F}_\mathrm{j}\left(\mathrm{X}_\mathrm{j}\right)}{\partial\mathrm{X}_\mathrm{j}}=\ \mathrm{C}\left(\mathrm{F}_1\left(\mathrm{X}_1\right),\ \ldots,\ \mathrm{F}_\mathrm{n}\left(\mathrm{X}_\mathrm{n}\right)\right)\times\prod_{\mathrm{j}=1}^{\mathrm{n}}\mathrm{f}_\mathrm{j}\left(\mathrm{X}_\mathrm{j}\right)$$

where f_j is the marginal density function, F_j (X_j) is the marginal distribution function, and c is the copula density function(Cherubini et al., 2004).

Copula functions allow modeling for the dependence structures and marginal distributions, providing flexibility in characterizing dependence. Some copula functions can also capture the tail dependence, that is, the probability that two variables experience extreme upward or downward movement. The upper and lower tail dependence can be obtained by the following:

$$\lambda_U=\lim_{u\to1}\frac{1-2u+C\left(u,u\right)}{1-u}$$

$$\lambda_L=\lim_{u\to1}\frac{C\left(u,u\right)}{u}$$

where $\lambda_U, \lambda_L \in [0, 1]$. If $\lambda_L > 0$ $(\lambda_U > 0)$, there is a lower (upper) tail dependence.

To obtain more accurate dependence characteristics, seven copula functions with different tail dependence characteristics are considered in this study. These copula functions are specified as follows.

3.2.1. Normal Copula (Gaussian)

The function of this Copula is as follows:

$$C_N\left(u,v;\rho\right)=\Phi\left(\Phi^{-1}\left(u\right),\Phi^{-1}\left(v\right)\right)$$

In this equation $\rho \in [-1,1]$ is the correlation parameter. Φ is the Gaussian standard cumulative distribution function and $\Phi^{-1}(u)$, $\Phi^{-1}(v)$ are the standard function. The tail dependence of this Copula function is zero.

$$\Phi_p\left(\Phi_{u_1}^{-1},\ldots,\Phi_{u_2}^{-1}\right)=\int_{-\infty}^{\Phi_{u_1}^{-1}}\cdots\int_{-\infty}^{\Phi_{u_n}^{-1}}\frac{1}{2\pi^{\frac{\pi}{2}}\left|R\right|^{\frac{1}{2}}}\exp(-\frac{1}{2}X^TR^{-1}X)dx_1\ldots dx_n$$

Given that in Gaussian Copula, the marginal density functions are considered to be the standard normal, the Gaussian Copula density is obtained as (Cherubini et al, 2004):

$$\frac{1}{\left(2\pi\right)^{\frac{\pi}{2}}\left|\mathrm{R}\right|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}X^T\mathrm{R}^{-1}X\right)=C^N\left(\Phi_{u_1},\ldots,\Phi_{u_n}\right)\times\prod_{i=1}^{n}(\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}x_i^2))$$

So we will have the following:

$$C^N(u_1,\dots,u_n)=\frac{1}{|\rho|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}\delta^T\left(\rho^{-1}-1\right)\delta\right)$$

where $\delta=\left(\Phi_{u_1}^{-1},\ \dots,\ \Phi_{u_n}^{-1}\right)^T$.

3.2.2. Student-t Copula

Student-t copula function is defined as:

$$C_T(u,v;\rho)=T\left(t_\upsilon^{-1}(u),t_\upsilon^{-1}(v)\right)$$

In which υ is the parameter of degree-of-freedom, $t^{-1}(u)$ and $t^{-1}(v)$ are the quantile functions of Student-t, and T is the bivariate Student-t cumulative distribution function. This copula function has symmetric nonzero tail dependence, and the tail dependence can be obtained by:

$$\lambda_U=\lambda_L=2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$$

The density function and the cumulative distribution function of the t-student distribution are:

$$t_v(x)=\int_{-\infty}^{x}\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{s^2}{v}\right)^{-\frac{v+1}{2}}ds$$

$$T_{v,\rho}(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}\frac{1}{2\pi\sqrt{1-\rho^2}}(1+\left(\frac{s^2+t^2-st\rho}{v\left(1-\rho^2\right)}\right)^{-\frac{v+2}{2}}dsdt$$

3.2.3. Clayton Copula

Clayton copula function is defined as:

$$C_{CL}(u,v;\delta)=\max\left\{\left(u^{-\delta}+v^{-\delta}-1\right)^{\frac{-1}{\delta}},0\right\}$$

It allows for lower tail dependence and upper tail independence:

$$\lambda_L=2^{\frac{-1}{\delta}},\ \delta>-1\qquad\text{and}\qquad\lambda_U=0$$

The density function of this Copula is as follows:

$$c^{CL}(u,\ v)=(\delta+1)(u^{-\delta}+v^{-\delta}-1)^{-\frac{2}{\delta}}(uv)^{-(\delta-1)}$$

3.2.4. Rotated Clayton Copula

Rotated Clayton copula is defined as:

$$C_{RCL}(u,v;\delta)=u+v-1+C_{CL}(1-u,1-v;\delta)$$

This copula has an upper tail dependence and no lower tail dependence:

$$\lambda_U=2^{\frac{-1}{\delta}},\ \delta>-1$$

3.2.5. Gumbel Copula

Gumbel copula is defined as:

$$C_G(u,v;\delta)=\exp\left\{-\left((-\log u)^{\delta}+(-\log v)^{\delta}\right)^{\frac{1}{\delta}}\right\}$$

The tail dependence of this copula function is asymmetric with lower tail independence and upper tail dependence:

$$\lambda_U=2-2^{\frac{1}{\delta}},\ \delta>1$$

3.2.6. Rotated Gumbel Copula

Rotated Gumbel copula is defined as:

$$C_{RG}\left(u,v;\delta\right)=u+v-1+C_G\left(1-u,1-v;\delta\right)$$

Unlike the Gumbel Copula, this Copula function has a lower tail dependence but no upper tail dependence.

$$\lambda_L=2-2^{\frac{1}{\delta}},\ \ \delta>1$$

3.2.7. Frank Copula

Frank copula was introduced by Frank in 1979. This Copula is the symmetric state of Archimedean Copula. The density function and the cumulative distribution function of this Copula are as follows:

$$c^F\left(u,\,v\right)=\frac{\theta e^{-\theta(u+v)}\left(e^{-\theta}-1\right)}{\left(e^{-\theta(u+v)}-e^{-\theta u}-e^{-\theta v}+e^{-\theta}\right)^2}$$

$$C^F\left(u_1,\,u_2\right)=-\frac{1}{\theta}\ln\left(1+\frac{\left(e^{-\theta u_1}-1\right)\left(e^{-\theta u_2}-1\right)}{e^{-\theta}-1}\right)$$

In this Copula function, the dependence on the upper tail and the lower tail is zero.

In the following, Table 1 shows a summary of the copulas used in this article, including Elliptical(Normal, t-student) and Archimedean(Frank, Clayton, Gumbel) copulas.

Table 2 shows a summary of the BB copula family that is also used in this article. Each member of the BB family is a combination of two Archimedean copulas. Therefore, the BB family can be included in the family of Archimedean copulas. Copula BB1 is a combination of Clayton and Gumbel copulas, BB6 is a combination of Joe and Gumbel copulas, BB7 is a combination of Joe and Clayton copulas, and BB8 is a combination of Joe and Frank copulas(Brechmann & Ulf Schepsmeier, 2013).

Table 1. Summary of copulas

	$C_N(u,v;\rho)=\Phi(\Phi^{-1}(u),\Phi^{-1}(v))$		Normal
$\rho\ \epsilon\ [-1,1]$	Tail dependence is zero		$Kendall=\frac{2}{\pi}\sin^{-1}\rho$
	$C_T(u,v;\rho)=T(t_\upsilon^{-1}(u),t_\upsilon^{-1}(v))$		t-student
$\rho\ \epsilon\ [-1,1]$	$\lambda_U=\lambda_L=2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$		$Kendall=\frac{2}{\pi}\sin^{-1}\rho$
	$C_{CL}(u,v;\delta)=\max\left\{\left(u^{-\delta}+v^{-\delta}-1\right)^{\frac{-1}{\delta}},0\right\}$		Clayton
$\delta\ \epsilon\ [-1,\infty)$	$\lambda_L=2^{\frac{-1}{\delta}},\quad \delta>-1$	Lower tail dependence	$Kendall=\frac{\delta}{\delta+2}$
	$C_{RCL}(u,v;\delta)=u+v-1+C_{CL}(1-u,1-v;\delta)$		Rotated Clayton
$\delta\ \epsilon\ [-1,\infty)$	$\lambda_U=2^{\frac{-1}{\delta}}$	Upper tail dependence	
	$C_G(u,v;\delta)=\exp\left\{-\left((-\log u)^\delta+(-\log v)^\delta\right)^{\frac{1}{\delta}}\right\}$		Gumbel
$\delta\ \epsilon\ [1,\infty)$	$\lambda_U=2-2^{\frac{1}{\delta}}$	Upper tail dependence	$Kendall=1-\frac{1}{\theta}$
	$C_{RG}(u,v;\delta)=u+v-1+C_G(1-u,1-v;\delta)$		Rotated Gumbel
$\delta\ \epsilon\ [1,\infty)$	$\lambda_L=2-2^{\frac{1}{\delta}}$	Lower tail dependence	
	$C^F(u_1,u_2)=-\frac{1}{\theta}\ln\left(1+\frac{\left(e^{-\theta u_1}-1\right)\left(e^{-\theta u_2}-1\right)}{e^{-\theta}-1}\right)$		Frank Copula
$\theta\ \epsilon\ R-\{0\}$	Tail dependence is zero		$Kendall=1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$

Table 2. Summary of BB copula family

	Generator function	Par	Kendall	Tail dependence
BB1 (Clayton-Gumbel)	$\left(t^{-\theta}-1\right)^{\delta}$	$\theta \geq 0$, $\delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$\left(2^{\frac{1}{\delta\theta}}, 2-2^{\frac{1}{\delta}}\right)$
BB6 (Joe-Gumbel)	$\left(-\log(1-t)^{\theta}\right)^{\delta}$	$\theta \geq 1$, $\delta \geq 1$	$1+\frac{4}{\delta\theta}\int_0^1\left(-\log(1-t)^{\theta}\right)\times\left(1-t\right)(1-(1-t)^{\theta}\right).dt$	$\left(0{,}2-2^{\frac{1}{\delta\theta}}\right)$
BB7 (Joe-clayton)	$\left(1-(1-t)^{\theta}\right)^{-\delta}-1$	$\theta \geq 1$, $\delta \geq 0$	$1+\frac{4}{\delta\theta}\int_0^1(-\left(1-(1-t)^{\theta}\right)^{\delta+1}\times\frac{\left(1-(1-t)^{\theta}\right)^{-\delta}-1}{(1-t)^{\theta-1}}).\mathrm{dt}$	$\left(2^{\frac{-1}{\delta}}, 2-2^{\frac{1}{\theta}}\right)$
BB8 (Joe-frank)	$-\log\frac{1-(1-t\delta)^{\theta}}{1-(1-\delta)^{\theta}}$	$\theta \geq 1$, $\delta \in 1]$	$1+\frac{4}{\delta\theta}\int_0^1(-\log\frac{(1-t\delta)^{\theta}-1}{(1-\delta)^{\theta}-1})\times(1-t\delta)(1-(1-t\delta)^{-\theta}).\mathrm{dt}$	(0,0)

3.3. Markov-Switching Copula

The time-varying copulas have been introduced by (Patton, 2006) to allow for time-variation in the dependence structure. They constitute an extension of Sklar's theorem, which shows that any joint distribution function may be decomposed into its marginal distributions and a copula that describes the dependence between the variables, for the conditional case. In what follows, we give a general definition of the conditional copula and present the time-varying copula functions used to examine the dependence between the series over time. We consider several time-varying copulas that capture different patterns of dependence, namely, Normal, t-Student, Gumbel, Clayton and Rotated Clayton, and Rotated Gumbel. The Gaussian and t-Student are characterized by symmetric dependence, while the Gumbel and Clayton are used to capture the right and the left dependencies respectively.

The conditional copula C is the joint distribution function of $F_{X_1|H}(X_1|H)$ and $F_{X_2|H}(X_2|H)$, where these two are the conditional marginals of X_1 and X_2 given a conditioning variable H.

3.3.1. Sklar's theorem

Let $F_{X_1X_2|H}(x_1,x_2|H)$ be the bivariate conditional distribution of $(X_1,X_2|H)$ with continuous conditional marginals $F_{X_1|H}(X_1|H)$ and $F_{X_2|H}(X_2|H)$. Then, there is a unique conditional copula C such that:

$$F_{X_1X_2|H}(x_1,x_2|H)=C\left(F_{X_1|H}(X_1|H),F_{X_2|H}(X_2|H)\right)$$

To model the joint conditional distribution, the evolution of the conditional copula C has to be specified and the functional form of C is fixed (Patton, 2006).

In this paper, we assume that the dependence parameter is allowed to vary over time and follows a restricted ARMA(1,1) process where the intercept term switches according to some homogeneous Markov process. However, we consider $S_t \rightarrow$ Markov (P), where S_t is a Markov chain irreducible and ergodic with n possible state spaces, i.e, P is a n*n matrix for these states and the transition probabilities in matrix P are equal to $p_{ij}=P(S_t=j|S_{t-1}=i)$ where $\sum_{j=1}^{n}p_{ij}=1$ for all of the time. p_{ij} is the probability of being in regime i at time t given that the market was in regime j at time t −1. For instance, if n=2 then:

$$P=\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

4. Empirical results

4.1. Data and summary statistics

The data set of the present study includes 1177 price data and returns of the five major cryptocurrencies, including Bitcoin-Ethereum-Ripple-Binance coin-Cardano, and Gold and Oil, in the period from April 26, 2018, to July 15, 2021. The selected period allows us to consider the period of recession before 2021 as well as the period of growth in 2021.

Table 3 describes the statistics for each time series. In this table, it can be seen that according to the standard deviation, Ripple currency has the highest risk (6.34), followed by Cardano with (6.04) and the lowest risk belongs to Gold. It can also be noted that the risk of Bitcoin is lower than all other currencies and close to the risk of Oil. Also, the highest average return belongs to Binance coin (44%) and the lowest belongs to Gold (7.9%). It is important to note that the Ripple currency has the highest risk and lowest returns among other cryptocurrencies, and the reason for this can be found in the litigation in which the cryptocurrency is involved in the courts of the United Kingdom. Gold, Bitcoin, and Ethereum also have negative skewness (skew to the right), and other assets have positive skewness. Also, all the assets under study have higher kurtosis than the normal distribution. The Jarque-Bera test also strongly rejects the null hypothesis that time series are normal for all time series.

Table 4 shows the matrix of correlation coefficients between the studied data. As can be seen, in the period under study, the highest correlation coefficients of bitcoins are with Cardano, Binance coin, Ripple, Ethereum, Gold, and finally Oil, respectively. A noteworthy point is the correlation coefficient between Bitcoin and Ethereum in this table (about 20%) which shows that the Ethereum currency does not follow Bitcoin entirely as in the past.

Table 3. Summary statistics

Jarqu-bra	Kurt	Skew	Std.dev	3th Q	1th Q	Min	Max	Median	Mean	
1236	10/206	-0/516	3/874	1/78	-1/53	-38/1	19/56	0/13	0/186	BTC
483	7/495	-0/357	5/243	2/77	-2/13	-44/7	26/46	0/12	0/238	ETH
4114	15/914	1/548	6/340	2/17	-2/30	-41/78	56/67	0	0/171	XRP
8376	21/758	1/347	5/943	2/97	-2/09	-43/96	69/97	0/12	0/441	BNB
59	4/489	0/289	6/043	3/00	-2/90	-41/72	33/22	0	0/311	ADA
274	6/426	-0/085	1/286	0/39	-0/31	-7/48	7/41	0/09	0/079	GOLD
15446	28/733	0/013	3/107	1/00	-0/82	-24/59	25/1	0/135	0/084	OIL

Table 4. Correlation coefficients

XRP	GOLD	OIL	BNB	ADA	ETH	BTC	
						1	BTC
					1	0/2029	ETH
				1	0/1906	0/7085	ADA
			1	0/6387	0/2025	0/6662	BNB
		1	-0/00413	0/0150	0/0179	0/0054	OIL
	1	0/0164	0/0785	0/0789	-0/0171	0/1098	GOLD
1	0/0348	0/011079	0/5586	0/6383	0/194	0/599	XRP

Note: Correlation coefficients are numbers that vary in the range (-1,1). A value of 1 indicates an absolute direct correlation and a value of -1 indicates an absolute inverse correlation.

4.2. Model estimation

Before estimating the marginal distribution of stock returns, some necessary diagnoses need to be tested. Table 5 presents the autocorrelation, conditional heteroscedasticity tests, Normal test, and stationary test for all asset returns. In the first part of Table 5, the results of the Shapiro test are given, which strongly rejects the Null hypothesis, which is the normality of the time series of this study. In the second part of Table 5, The Dickey-Fuller test shows that in the all-time series studied, the null hypothesis that the time series is non-stationary is rejected and the series remains stationary.

The Ljung -Box test was used to examine the autocorrelation between the data of each time series. As can be seen in the third section of Table 5, at all levels of significance it rejects the null hypothesis that there is no autocorrelation between data over time for all assets except Ripple.

The ARCH effect is related to heteroscedasticity, often referred to as the serial correlation of variances. This effect is often manifested when a variance or fluctuation of a particular variable creates a pattern that is determined by certain factors. In the fourth section of Table 5, it is shown that in the time series of the assets of Cardano, Binance coin, Ripple, Oil, and Gold, the null hypothesis that there is no ARCH effect is rejected and their time series has variance heterogeneity. But Bitcoin and Ethereum are found to have no arch effect.

Table 6 presents the estimated coefficients of marginal distribution for each asset return based on the ARMA(1, 1)-GARCH(1, 1)-t model.

Table 7 shows the results of the Ljung-Box test after implementing the GARCH model. The results show that the effect of autocorrelation on time series data is eliminated.

Table 5. Autocorrelation and conditional heteroscedasticity tests

BTC	ETH	ADA	BNB	OIL	GOLD	XRP	
SHAPIRO TEST							
0. 888	0. 9	0. 94	0. 867	0. 724	0. 82	0. 828	Statistic
0. 000*	0. 000*	0. 000*	0. 000*	0. 000*	0. 000*	0. 000*	P-value
The Dickey-Fuller test							
-16. 1	-16. 45	-16. 35	-15. 95	-19. 82	-19. 33	-16.5	Statistic
0. 01**	0. 01**	0. 01**	0. 01**	0. 012**	0. 01**	0.01**	P-value
The Ljung -Box test							
20. 44	34. 75	22. 23	20. 68	14. 43	61. 2	2. 13	Statistic
0. 0001***	0. 00***	0. 000***	0. 0001***	0. 002***	0. 000***	0. 54	p-value
The ARCH effect test							
0.53	2. 66	8. 26	20. 78	189	59. 2	27.45	Statistic
0.466	0. 1026	0. 004****	0. 00****	0. 00****	0. 00****	0.00****	p-value

Note:
* Means rejecting the null hypothesis, which indicates that the distribution is not normal.
** Means that the null hypothesis of this test is rejected and the series is stationary.
*** Indicates that the null test hypothesis that there is no autocorrelation is rejected.
**** Indicates that the null hypothesis is rejected and there is an ARCH effect.

Table 6. Coefficient estimation of marginal distribution for each asset return

	BTC	**ETH**	**ADA**	**BNB**	**OIL**	**GOLD**	**XRP**
Conditional mean							
Constant	0. 0008 (0. 001)	-0. 00001 (0. 001)	0. 0002 (0. 0017)	0. 0005 (0. 0015)	0. 0002 (0. 0006)	-0. 00018 (0. 0002)	-0. 0022 (0. 0017)
AR(1)	-0. 417 (0. 16)	-0. 605 (0. 126)	-0. 527 (0. 098)	-0. 63 (0. 277)	-0. 479 (0. 1458)	-0. 36 (0. 142)	-0. 53 (0. 24)
MA(1)	0. 315 (0. 18)	0. 512 (0. 132)	0. 435 (0. 1)	0. 56 (0. 298)	0. 374 (015)	0. 129 (0. 166)	0. 445 (0. 236)
Conditional variance							
OMEGA	0. 00011 (0. 00004)	0. 00018 (0. 00009)	0. 0003 (0. 0001)	0. 0001 (0. 00006)	0. 0001 (0. 0002)	000002 (0. 000006)	0. 0005 (0. 0002)
ALPHA	0. 082 (0. 048)	0. 079 (0. 0397)	0. 106 (0. 039)	0. 1532 (0. 053)	0. 11 (0. 07)	0. 55 (0. 1339)	0. 475 (0. 173)
BETA	0. 851 (0. 046)	0. 86 (0. 052)	0. 794 (0. 055)	0. 8216 (0. 0465)	0. 872 (0. 05)	0. 356 (0. 054)	0. 5237 (0. 1426)
Skew	0. 922 (0. 067)	0. 894 (0. 049)	0. 9568 (0. 058)	0. 8917 (0. 052)	0. 855 (0. 041)	0. 87 (0. 043)	0. 976 (0. 049)
Log-like	2365. 13	1970. 6	1768. 8	1937. 96	3074. 4	4033. 8	2015. 98

Note:
() indicates standard error.

Table 7. Ljung-box test after using the GARCH model

BTC	**ETH**	**ADA**	**BNB**	**OIL**	**GOLD**	**XRP**	
0. 07	0. 0247	0. 06	0. 67	0. 73	0. 88	0. 45	Statistic
0. 79	0. 87	0. 8	0. 41	0. 39	0. 2	0. 5	p-value

In this study, the Copula approach was used to find the joint-distribution function between each pair of assets. To evaluate how the models used in this research perform compared to other Copula models, 11 different Copula models and 6 Hidden Markov Switching Copula models were used.

Therefore, initially, all the copulas were used through the recursive method for each asset pair, and log-likelihood values were obtained for each model. Then the best model was selected for each pair.

Table 8 shows log-likelihood values for the selected time-varying Copulas with regime change, including MS-CN[1], MS-CT[2], MS-CC[3], MS-CG[4], MS-CRC[5], and MS-CRG[6], as well as log-likelihood values For other copulas, it shows Normal, t-student, Clayton, frank, joe, Rotated Clayton, BB8, BB7, BB6, BB1, Gumbel, and Rotated Gumbel. From Table 8 it can be seen that all log-likelihood values for time-varying Copulas with regime change are obviously more significant than the static Copula values for all asset pairs.

Table 8. The log-likelihood value for time-varying copula with regime switching.

	BTC-ETH	BTC-ADA	BTC-BNB	BTC-OIL	BTC-GOLD	BTC-XRP
MS-CN	81.6	510.18	426.1	**3.65**	8.43	500
MS-CT	**91.15**	**528.18**	437.7	3.45	**9.24**	443
MS-CC	90.7	512.8	445.5	1.3	5.28	498
MS-CG	69.7	354.2	282	2.53	5.05	443.6
MS-CRC	37.5	243	196	2.68	5.68	357
MS-CRG	36.4	289.9	**468.2**	0.9	5.58	**538**
normal	25. 21	396. 5	337	0	7. 85	361. 8
T student	41. 67	468. 4	373. 2	2. 16	7. 43	440. 2
clayton	27. 16	467. 9	386. 5	0. 51	5. 28	391. 2
gumbel	26. 71	354. 7	382. 15	0. 08	5. 05	339. 1
frank	21	398. 2	328. 9	0. 17	7. 76	395. 71
joe	19. 82	231. 3	177. 9	0. 01	2. 9	228. 65
BB1 (clayton-Gumbel)	33. 99	489. 4	401. 3	0. 5	6. 34	424. 4
BB6 (joe-Gumbel)	26. 7	354. 6	282	0. 07	5. 03	339
BB7 (Joe-clayton)	34. 7	486. 5	398. 8	0. 5	5. 94	413. 6
BB8 (joe-frank)	22. 15	358	297. 5	0	7. 85	362. 3
RC	20. 52	243	196. 9	0. 21	5. 63	240. 7

As shown in Table 8, the MS-CT model is best for the BTC-ETH, BTC-ADA, and BTC-GOLD pairs, the MS-CRG model is the best model for the BTC-XRP and BTC-BNB pairs, and the MS-CN model is the best model for the BTC-OIL pair.

Table 9 summarizes the parameters obtained from the best time-varying Copula model with regime switching for each asset pair. This paper uses the MLE method to estimate the Copula model based on (Joey 1997). In this section, we distinguish two modeled regimes into two modes of high dependency and low dependency.

Table 9. Coefficient estimation for time-varying copula with regime switching.

	ω_1	df_1	ω_2	df_2	$Kendall_{reg1}$	$Kendall_{reg2}$	P
BTC-ETH (MS-CT)	0. 041 (0. 039)	9. 85 (4. 25)	0. 68 (0. 047)	3. 07 (0. 99)	0. 477	0. 026	$\begin{pmatrix} 0.999 & 0.001 \\ 0 & 1 \end{pmatrix}$
BTC-ADA (MS-CT)	0. 884 (0. 014)	3. 2 (0. 8)	0. 5 (0. 047)	7. 5 (2. 94)	0. 69	0. 337	$\begin{pmatrix} 0.968 & 0.032 \\ 0.042 & 0.958 \end{pmatrix}$
BTC-BNB (MS-CRG)	3. 244 (0. 2)	-	1. 59 (0. 059)	-	0. 69	0. 37	$\begin{pmatrix} 0.976 & 0.024 \\ 0.015 & 0.985 \end{pmatrix}$
BTC-OIL (MS-CN)	-0. 66 (0. 100)	-	0. 111 (0. 036)	-	-0. 46	0. 07	$\begin{pmatrix} 0.011 & 0.989 \\ 0.166 & 0.834 \end{pmatrix}$
BTC-GOLD (MS-CT)	0. 062 (0. 04)	30 (28. 3)	0. 226 (0. 06)	10. 51 (7. 75)	0. 04	0. 145	$\begin{pmatrix} 0.998 & 0.002 \\ 0.004 & 0.996 \end{pmatrix}$
BTC-XRP (MS-CRG)	3. 55 (0. 195)	-	1. 51 (0. 06)	-	0. 718	0. 34	$\begin{pmatrix} 0.975 & 0.025 \\ 0.027 & 0973 \end{pmatrix}$

1. Markov switching-copula normal model
2. Markov switching-copula t-student model
3. Markov switching-copula clayton model
4. Markov switching-copula Gumbel model
5. Markov switching-copula rotated clayton model
6. Markov switching-copula rotated Gumbel model

In Table 9, the values ω_1 and ω_2 are the Copula parameters for each pair from regime one and regime 2, and the values df_1 and df_2 are the degrees of freedom for those pairs that use the MS-CT model for regime one and regime 2. $Kendall_{reg1}$ is the Kendall-Tau correlation coefficient for regime 1 and $Kendal_{reg2}$ for regime 2. The last column also shows the P matrix, which shows the transition matrix between different regimes over time. For example, for the BTC-ADA pair, the value of P_{11} is 0.968, which indicates the probability of the presence of the pair in regime 1, and the value of P_{22} is 0.958, which indicates the probability of the relationship of this pair in regime 2. However, it is better to analyze the transfer matrix column more in Table 9.

In Table 9, for the BTC-ETH pair, it can be seen that the Kendall-Tau coefficient of regime 1 is 0.477, which we consider to be a high-dependent state, and the Kendall-Tau coefficient of regime 2 is 0.026, which makes this regime a low-dependent state. So, when the relationship between these two pairs is in regime 1, that period is most likely the period when the dependence between these pairs is high and therefore their movement with each other is maximal.

In Table 9, for the BTC-ADA, BTC-BNB, and BTC-XRP pairs, the Kendall-Tau values of regime 1 are greater than those of regime 2, and therefore, for these three pairs, regime 1 is highly dependent, and regime 2 is low dependent. Also, for the BTC-GOLD pair, it can be seen that the Kendall-Tau regime 2 is more than 1, so we consider the second regime to be highly dependent and regime 1 to be low dependent. For the BTC-OIL pair, there is a noteworthy point. It can be seen that the Kendall-Tau coefficient of regime 1 is negative, which we consider as a high dependence state, and regime two as a low dependence state. It is later stated what the relationship is for this pair over time in terms of presence in time-varying regimes.

4.3. Smoothed probabilities

The two dependency regimes defined in the previous section can now be considered with smooth probabilities. Figures 1 to 6 show the probability of each regime in a high or low dependence on variable time for each yield pair. At each point in time, the sum of the probabilities of high dependency and low dependency is equal to 1, but the probability of change between regimes (high and low dependency) varies for each pair over time.

As can be seen in Figure 1, the regime change in the BTC-ETH pair relationship did not occur much, and over time, the model can predict what the relationship will be between them. The upper diagram in Figure 1 shows regime one, and the lower diagram shows regime two and shows the probabilities of entering each regime. In this figure, it can be seen that Markov's replacement model characterizes the regime established between these pairs up to the 980 period of regime 1, which enters the second regime (high dependency mode) approximately after the 980 period. It can be seen that according to the price chart of Bitcoin and Ethereum, this regime change occurred during the bullish market.

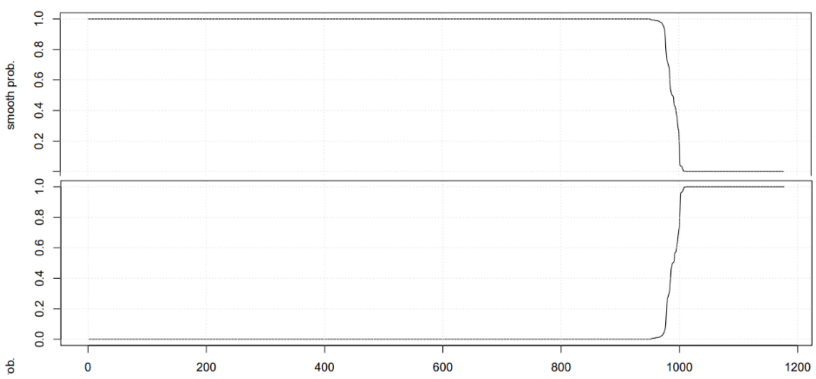

Fig. 1. Smoothed probabilities for low and high dependence regimes BTC-ETH

Figure 2 shows the regime changes for the BTC-ADA pair. As we can see, the regime changes in this pair are relatively greater than the BTC-ETH pair, but it can be seen that over time there have been many periods in which the correlation between Bitcoin and Cardano has been minimized and entered into the regime 2. But in general, most of the time the relationship between this pair was in regime 1.

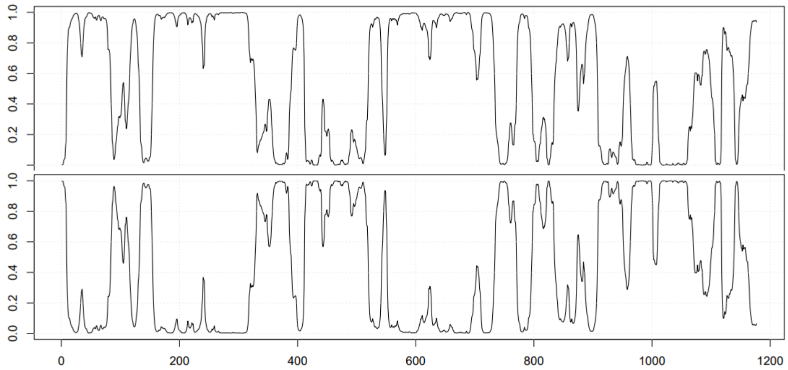

Fig. 2. Smoothed probabilities for low and high dependence regimes BTC-ADA

Figure 3 shows the probabilities of smoothness between the BTC-BNB pair. It is observed that in the period of 500 to 800, the relationship between the two is in the first regime, and in the bullish market, the relationship between the two currencies is mainly in the second regime. But what is the cause?

As can be seen in Figure 3, from the 900 period, when the market was highly bullish, the relationship of Bitcoins to these currencies, namely Cardano and Binance coins, fell into regime 2(low dependency). This is due to the 40-fold increase in Binance coin and the 30-fold increase in Cardano versus the 7-fold increase in Bitcoin. Therefore, it can be noted that if the market is bullish and the dependency regime enters a state of low dependence, it can be expected that the second currency (here Binance coin and Cardano) will grow faster than Bitcoin. This can also be a topic for research in future articles.

In Figure 4 we see the relationship between the BTC-XRP pair, which is almost similar to the relationship between Bitcoin and Cardano.

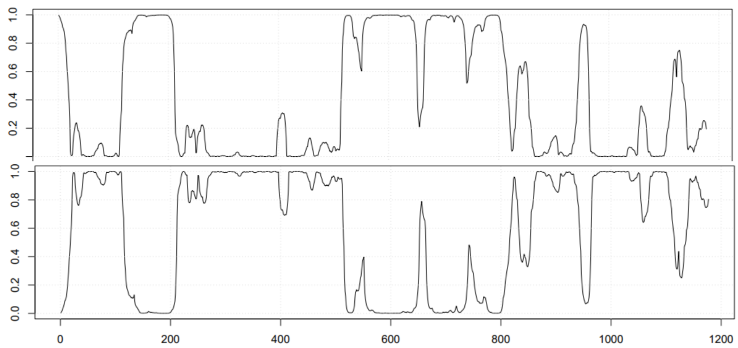

Fig. 3. Smoothed probabilities for low and high dependence regimes BTC-BNB

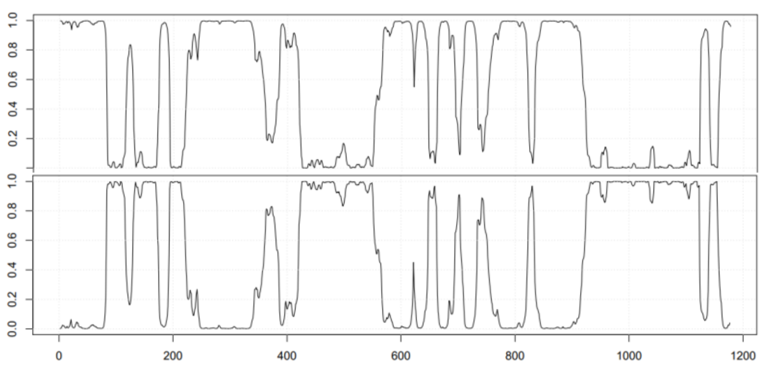

Fig. 4. Smoothed probabilities for low and high dependence regimes BTC-XRP

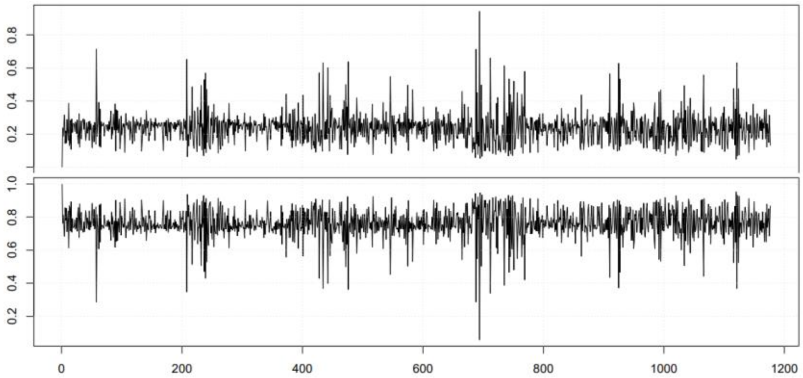

Fig. 5. Smoothed probabilities for low and high dependence regimes BTC-OIL

Figure 5 shows that the probability of the BTC-OIL pair being in different regimes is highly variable and fluctuates. But with more attention, it can be seen that the main situations of the relationship between these two currencies are in the second regime, and in most cases, the probability of being in this regime is close to 80%. This issue can be analyzed from the relationship transfer matrix of this pair. As shown in the following matrix:

$$\begin{pmatrix} 0.011 & 0.989 \\ 0.166 & 0.834 \end{pmatrix}$$

This matrix shows that the probability of being in regime 1 is very low, close to 1%, and after entering regime 1, one can expect to enter regime 2 with a very high probability. Also, the probability of being present in the second regime is about 80% and it is 20% probable that it will enter the first regime from the second regime, which is due to the large fluctuations in the positions between these two financial assets.

Finally, Figure 6 shows the relationship between the Gold and Bitcoin regimes in a period (nearly a year), the state between the two assets enters regime 2 (high dependence) and then returns to the first regime. Therefore, as can be seen from the transfer matrix of this pair in Table 9, the placement in different states between these two assets over time is relatively stable, and if the relationship between the two enters regime 2 (high dependence), it can be analyzed as follows: Most likely it will stay in this state for a long time.

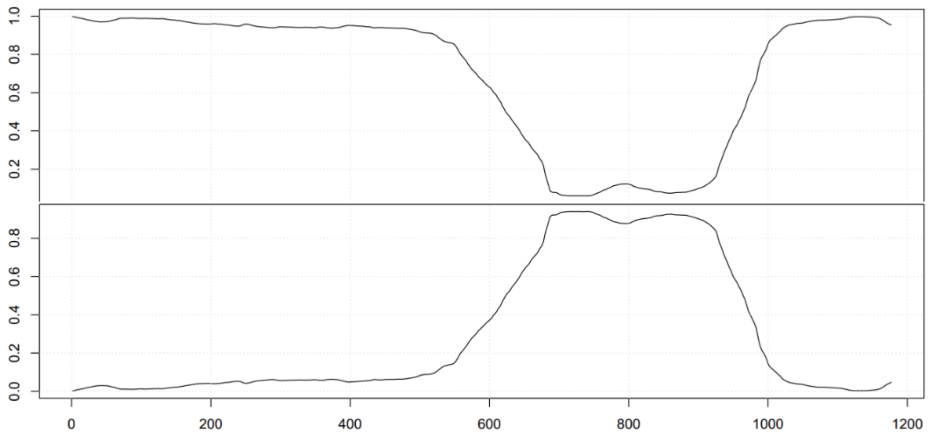

Fig. 6. Smoothed probabilities for low and high dependence regimes BTC-GOLD

5. Conclusions

As the literature points out, the structure of dependencies between financial assets varies over time, and it is important to choose an appropriate model to examine this structure of dependencies over time. In the past, the value of this dependence was considered constant over time, but over time, different models were presented and showed that the correlation parameter can be obtained over time. Copula's method is one of these methods, which determines this by constructing a joint-distribution function between pairs of assets. Also, the Markov switching model, due to its memoryless nature, examines the issue of which mode of dependence it is in at any given time. That is, by combining this model with the Copula model, it can be concluded that at any time, it is possible to determine the relationship between different pairs of assets.

This paper analyses the dependence structure and the regime change between Bitcoin and six other assets, including Gold and Oil, using six time-varying copula models with Markov switching (including MS-CC, MS-CT, MS-CN, MS-CG, MS-CRG, MS-CRC) and 11 separate copula model while (Niu, Xu & Xiong, 2023) only used a Markov-switching mixed-Clayton model(MS-CC) to check the correlation structure and regime changes. As we can see in this paper, the MS-CC model is not the optimal model for any of the investigated couples, and this can be because the data of the two papers are different or are analyzed at different times.

Our models reveal the dynamic dependence structure of the Crypto markets with two different regimes. Generally, the dependencies between the BTC and the six other assets are positive. Two distinct dependence regimes, named high dependence and low dependence, are verified by different dependence parameters across regimes. Results show that for Bitcoin-Ethereum, Bitcoin-Cardano, and Bitcoin-Gold pairs model MS-CT, for Bitcoin-Binance coin and Bitcoin-Ripple pairs MS-CRG and MS-CN for Bitcoin-Oil pair have the best performance. These results are almost confirmed by (Ji, Liu, and Cunado, 2020), who used the combined model of Markov switching with copula to evaluate the risk spillover between stock exchanges of G7 member countries and for three pairs of these countries, the MS-CT model and for one MS-CN model was the optimal model to evaluate the correlation structure. Although the test data in the two papers are different, the results show that for different pairs, Markov switching models with copula have the best performance in any way and only their type changes for pairs. Some of these results can also be seen in (Zhu, Yamaka, and Sriboonchitta, 2016) , Unlike our paper (Gozgor, Tiwari, Khraief, 2019) used the Gumbel, the Rotated Gumbel, the Normal, the Student's t, and the Symmetrized Joe-Clayton models and the MS-CN model was optimized for their research.

The estimated smoothing probabilities imply that the dependence structure between the BTC and the other assets changes over time. The status of a dependence regime is persistent and highly correlated with changes in tail dependence. Moreover, there is a probability that dependence relations between the BTC and ETH will enter a high dependence regime in a bullish market.

It is also observed for the pairs of BTC-BNB, BTC-ADA, and BTC-XRP, which have entered regime 2 in periods, which in the bullish markets show the further ascent of these assets, and in the descending market as a sign of the further descent of these Altcoins. For the BTC-GOLD pair, it is also observed that they have a relatively stable relationship over time, and in the periods that they enter regime 2, they have a similar Co-movement with each other, and it is possible to predict the future movement of these two assets when they are in regime 2. Finally, for the BTC-OIL pair, it is observed that over time, they are generally in regime two and do not have a strong relationship with which to predict the future correctly. This is because the regime changes between the two assets are increasing and fluctuating.

Of course, the limitations of this method are important. To check the dependence structure and use the Markov switching model, there must be a reasonable relationship between the two studied assets. Therefore, in this paper, the correlation coefficient has been used to investigate this relationship.

For future studies, it is suggested that the above model be used to investigate the effect of risk spillover between different assets and to examine the effects of risk spillover as well as the symmetry in the upside and downside risk spillover. This can be an interesting subject when calculating future risk for each market.

Among the practical policies of this paper is to find the structure of dependence between financial assets and then use the Markov switching model to find different states between two assets so that it can predict the market trend for different assets and decide which one to Buy or sell a property.

Furthermore, from the policymaker's point of view, some phenomena such as spillover ,contagion, or co-movement between the prices of different commodities are as important as the business/economic cycles and regime switching; it is due to rising commodity prices simultaneously cause high inflation in economy (Ukraine Russia war impact on inflation). It is clear that dynamic dependency structure modeling can predict the mentioned phenomena with higher performance than static copulas, as elaborated in this paper. So, using a dynamic structure of dependency helps policymakers to better the decision-making process of the economy.

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